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Endogenous Market Structures and Strategic Trade Policy

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Abstract

We characterize the optimal export promoting policies for international markets whose structure is endogenous. Contrary to the ambiguous results of strategic trade policy for markets with a fixed number of firms, it is always optimal to subsidize exports as long as entry is endogenous, under both competition in quantities and in prices. With homogenous goods the optimal export subsidy is a fraction $1/\epsilon$ of the price, where $\epsilon$ is the elasticity of demand, the exact opposite of the optimal export tax in the neoclassical trade theory. A similar argument can be applied to show the general optimality of R&D subsidies and of competitive devaluations to promote exports in foreign markets where entry is endogenous.

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1 INTRODUCTION

A wide literature on optimal strategic trade policy and on other forms of strategic export promotion has been developed since the pathbreaking contributions of Brander and Spencer (1985), Eaton and Grossman (1986) and others. A disappointing result of this literature has been that its policy prescriptions on whether and how we should subsidize or tax exports have been largely ambiguous and dependent on the particular assumptions on the market structures under consideration, in particular whether competition is in quantities or prices (see Helpman and Krugman, 1989). This article argues that, independently from the assumptions on the market structures adopted in this literature, any ambiguity on the optimal unilateral export promoting policy vanishes under a single and (possibly) realistic condition. This condition is that entry of firms in the international competition is endogenous (i.e. determined by profit maximizing decisions of the firms). Under this condition, contrary to the traditional results, it is always optimal to subsidize exports under both competition in quantities and in prices. One can apply the same principle to general models of trade policy, R&D policy and exchange rate policy.

Common wisdom on the benefits of export subsidization largely departs from the implications of trade theory. While export promotion is often supported by governments, theory is hardly in favor of its direct or indirect implementation. In the standard neoclassical framework with perfect competition, the scope of trade policy is to improve the terms of trade, that is the price of exports relative to the price of imports, and, as long as a country is large enough to affect the terms of trade, it is optimal to tax exports (since this is equivalent to set a tariff on imports); more precisely, the optimal unilateral export tax can be derived as a fraction $1/\epsilon$ of the price, where $\epsilon$ is the elasticity of demand. The same outcome emerges under monopolistic competition, as shown by Helpman and Krugman (1989). In case of strategic interactions between few firms, however, a second aim of strategic trade policy is to shift profits toward the domestic firms, therefore a large body of recent literature has studied models with a fixed number of firms competing in a third market with positive profits. Here, the optimal unilateral policy is an export tax under price competition, or whenever

\footnote{Feenstra (2004) provides an updated review of this literature.}
strategic complementarity holds (Eaton and Grossman, 1986). Under quantity competition, an export subsidy can be optimal (Brander and Spencer, 1985), but only under certain conditions.\textsuperscript{2} According to a leading trade economist (Bhagwati, 1988), the ambiguity of these results “creates information requirements for policy intervention that appear to many of the architects of this theoretical innovation to be sufficiently intimidating to suggest that policymakers had better leave it alone”.\textsuperscript{3}

Nevertheless, different forms of direct or indirect export subsidies are widespread. Governments strongly support exporting firms, they often hide forms of export promotion behind nationalistic pride, and consider the conquer of larger market shares abroad as a positive achievement in itself. The European Union coordinates trade between its members and the rest of the world in a similar spirit, and subsidizes exports of agricultural products and the aircraft industry. France has often supported its “national champions” with public subsidies. Italy has a long tradition of public support of the Made in Italy, which is quite important for the promotion of fashion, design and food industries. Japan, Korea and other East-Asian countries have implemented export promoting policies for decades. Heavily protected South-American countries have tried to subsidize manufactured products in which they could develop a comparative advantage (and not only those). Even US has implemented strong forms of export subsidization through tax exemptions for a fraction of export profits, foreign tax credit\textsuperscript{4} and export credit subsidies.

It appears quite surprising that, in front of this, trade economists do not have clear and unambiguous arguments to explain why export subsidies could be the optimal unilateral trade policy. Building on the recent literature on endogenous market structures (see Etro, 2006, 2007a,b), this paper provides such an argument, studying a model of trade policy for a foreign market characterized by strategic interactions and endogenous entry of international firms.\textsuperscript{5}

\textsuperscript{2}These conditions are derived by Dixit (1984) and Klette (1994). See also Horstmann and Markusen (1986) for related results.

\textsuperscript{3}The literature has developed other arguments against export subsidies, as in case of asymmetric information between firms and government or in case of retaliation (see Brander, 1995 for reviews of the literature).

\textsuperscript{4}See Desai and Hines (2008).

\textsuperscript{5}Notice that free entry is a realistic assumption since a foreign country without a domestic
In general, a government may tax or subsidize domestic firms that are active in international markets for profit shifting reasons: the right policy allows the government to turn the domestic firm into a leader in the international competition, and to increase the net profits for the country. For instance, when a domestic firm competes against a foreign competitor in a third market, it is typically optimal to tax exports under competition in prices and to subsidize exports under competition in quantities: the reason is that in the former case a tax turns the domestic firm into an accommodating leader which softens price competition and earns higher profits, and in the latter case a subsidy turns the domestic firm into an aggressive leader which increases its production and earns higher profits as well. When entry in the international market is endogenous, the same general principle applies, but the only way for the domestic firm to earn positive profits is by adopting an aggressive strategy, either reducing prices or increasing production so as to conquer market shares and reduce average costs below those of the other firms (any accommodating strategy would end up attracting entry and profits would vanish). Therefore it is now unambiguously optimal to subsidize exports to turn the domestic firm into an aggressive leader under both competition in prices and quantities.

This article characterizes the optimal unilateral trade policy for a large class of models and analyzes a few examples. In the case of homogenous goods, U shaped cost functions and competition in quantities, the optimal unilateral export subsidy is a fraction $1/\epsilon$ of the price, where $\epsilon$ is again the elasticity of demand: notice that this is the exact opposite of the traditional neoclassical optimal policy. Product differentiation and competition in prices tend to reduce the optimal export subsidy.

The same argument can be applied to other forms of indirect export promotion, as policies which boost demand or decrease transport costs for the exporting firms, R&D subsidies for firms competing for global markets and exchange rate policy for firms active in foreign markets with endogenous entry: as long as these policies increase the marginal profitability of the domestic firm, there is a strategic incentive to use them unilaterally.

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firm in the market can only gain from allowing free entry of international firms. See Boone et al. (2006) for a related discussion. An important work that endogenizes the number of firms and of the exporting firms in a general equilibrium framework is Ghironi and Melitz (2005).
Finally, our result holds also in an equilibrium analysis where all the countries can choose optimally their trade policy and entry of firms in the international market is endogenous. We verify this in a model of competition in quantities with homogenous goods and increasing marginal costs, and we find out that the Nash equilibrium export subsidy remains the same as the unilateral optimal one for an endogenous number of countries (determined by the size of the international market), while the other countries commit to free trade.

The article is organized as follows. Section 2 introduces a general model and determines the strategic incentives to promote exports in the presence of an exogenous number of firms in the international market and with an endogenous market structure. Subsequently, it applies these general results to strategic trade policy, and derives the optimal unilateral export subsidies under competition in quantities and competition in prices. Section 3 discusses other applications: it studies the incentives to adopt R&D subsidies for domestic firms engaged in an international patent race, it studies the role of exchange rate policy in supporting exporters, and it extends the analysis of trade policy to an equilibrium set up in which multiple countries can choose their policy tools simultaneously. Section 4 concludes. Technical details are left in the Appendix.

2 THE MODEL

We will adopt a general model of market structures introduced in Etro (2006, 2008a), use it to describe competition in an international market and augment it by introducing export promoting policies.

Consider an international market where \( n \) firms from different countries are competing in Nash strategies. Let us assume that each firm chooses a strategic variable \( x_i \), with \( i = 1, 2, ..., n \), which delivers the net profit function:

\[
\pi_i = \Pi^i (x_i, \beta_i, s_i) - F
\]

where \( F \) is the fixed cost. The function \( \beta_i = \sum_{k=1,k\neq i}^{n} h(x_k) \) aggregates the strategies of the other firms, with \( h(\cdot) \) positive, differentiable and increasing. As we will see, the separability property that is assumed in the \( \beta_i \) function is satisfied by a large class of models of competition in quantities and in prices, and in other models as well. I assume that \( \Pi(x_i, \beta_i, s_i) \) is quasiconcave in \( x_i \) with
\( \Pi_{11} < 0. \) Since the main focus of the analysis will be on free entry equilibria, I assume that an increase in \( \beta_i \) reduces the profits of firm \( i: \Pi_2 < 0. \) In general \( \Pi_{12} \) could be positive, so that we have strategic complementarity, or negative so that we have strategic substitutability.

Finally, \( s_i \) is the export policy chosen by the government of country \( i: \) in our main application, this will be an export subsidy, but we will take in consideration other policies as well. Without loss of generality, an increase in the policy raises profits (\( \Pi_3 > 0 \)), therefore \( s_i \) will be defined as an export promotion policy for country \( i. \) I will allow \( \Pi_{13} \) to be positive or negative: only in the first case, the policy increases marginal profitability. As we will verify later, all forms of trade subsidies under quantity and price competition imply \( \Pi_{13} > 0, \) but other indirect forms of export promotion can be characterized by \( \Pi_{13} < 0. \)

In general, the welfare of country \( i \) depends positively on the profits of the domestic firm and negatively on the cost of its policy \( s_i, \) say \( C(s_i), \) so that we can express welfare as:

\[
W(s_i) = \Pi^i (x_i, \beta_i, s_i) - C(s_i) - F
\]  

(2)

In case of export subsidization, the cost of trade policy is the collection of tax revenue, but this may imply tax distortions or other kinds of costs due to general equilibrium or political considerations. Moreover, in the presence of lobbying activity, the weight given by the politicians to the costs of the policy may be variable. Finally, other forms of export promotion can have different costs for national welfare. Nevertheless, in line with the literature on strategic trade policy, our focus will be mainly on the strategic incentive to export promotion, which will be defined as the indirect marginal benefit of an increase in \( s_i \) on the profit:

\[
SI^i = \Pi^i_2 (x_i, \beta_i, s_i) \frac{\partial \beta_i}{\partial s_i}
\]  

(3)

As long as this is positive, the government of country \( i \) has a strategic reason to promote exports beyond any direct reason which depends on the first order impact of policy on welfare.\(^7\)

\(^{6}\)The subindex of the profit function refers to derivatives with respect to the corresponding argument.

\(^{7}\)In general, the optimal policy satisfies an optimality condition as \( \Pi^i_3 (x_i, \beta_i, s_i) + SI^i = \)
We will now present a few examples of market structures which are nested in the general model. As a first example let us consider models of competition in quantities. In particular, allowing for imperfect substitutability between goods, we can adopt an indirect demand for good $i$ as
$$p_i = p \left( x_i, \sum_{k=1, k \neq i}^n h(x_k) \right)$$
with $p_1 < 0$ and $p_2 < 0$; of course the case of homogenous goods is a particular case emerging when the inverse demand depends on the total production only,
$$p_i = p(X)$$
with $X = \sum_{k=1}^n x_k$. The cost function, which includes transport costs, can be expressed as $c(x_i)$ with $c'(\cdot) > 0$. It follows that, in the absence of any policy, the general expression for gross profits is given by:
$$\Pi^i(x_i, \beta_i, 0) = x_i p(x_i, \beta_i) - c(x_i)$$
(4)
where $\beta_i = \sum_{k=1, k \neq i}^n h(x_k)$.

As a second example let us consider a general class of models of price competition. Any model with direct demand $D_i = D \left[ p_i, \sum_{j=1, j \neq i}^n g(p_j) \right]$ where $D_1 < 0$, $D_2 < 0$, $g(p) > 0$ and $g'(p) < 0$, is nested in our general framework after adopting the monotonic transformation $x_i \equiv 1/p_i$ with $h(x) = g(1/x)$, so that $h'(x) = -g'(1/x)/x^2 > 0$. Under constant marginal costs, gross profits become:
$$\Pi^i(x_i, \beta_i, 0) = \left( \frac{1}{x_i} - c \right) D \left( \frac{1}{x_i}, \beta_i \right)$$
(5)
We will assume that strategic complementarity typically holds ($\Pi_{12} > 0$). As we will see later, examples include many well known demand functions like the class of demand functions derived by Dixit and Stiglitz (1977), the Logit demand and the class of demand functions with constant expenditure. An important case which is nested in this specification is the model of price competition with isoelastic demand, which has been widely employed in the new trade theory (Krugman, 1980; Helpman and Krugman, 1985).

In these basic models of the market structure we can introduce different policies for export promotion. In the rest of this section I will derive the general results in two crucial cases: in the first one the foreign market structure is exogenous, in the sense that there is a fixed number of firms (and I replicate $C^*(s_i)$, where the first and the last term represent the direct marginal benefits and costs of the policy.

8The condition for strategic complementarity is $D_2 + (p - c)D_{12} > 0$. 

7
the existing results in the literature), in the second one the foreign market is characterized by an endogenous structure, in the sense that entry is free or endogenous.

2.1 Strategic policy with exogenous market structures

Let us briefly summarize the results on the optimal unilateral trade policy for a foreign market with a fixed number of firms. More specifically, assume that \( s_i = 0 \) for all firms except the domestic one, whose policy \( s \) is chosen by the government of its home country at an initial stage. Consider the second stage after a policy \( s \) has been chosen and assume that a unique Nash equilibrium exists with the same strategy for the foreign firms, say \( x \), and a different strategy for the domestic one, say \( z \), depending on the policy \( s \). The first order equilibrium conditions are:

\[
\Pi_1 [x, (n - 2)h(x) + h(z), 0] = 0 \tag{6}
\]
\[
\Pi_1^H [z, (n - 1)h(x), s] = 0 \tag{7}
\]

which provide the equilibrium strategies of the domestic firm and of the international ones as functions of the policy. Changes in the domestic policy affect the strategies of all the firms. For instance, in case \( \Pi_{13}^H > 0 \) (which will always be the case when the policy is export subsidization), one can verify that an increase in \( s \) is always going to increase the domestic strategy \( z \), and to increase the strategy of the international firms \( x \) if and only if strategic complementarity \( (12 > 0) \) holds.

In the initial stage the government chooses the policy to maximize welfare taking these reactions into account. In the Appendix we derive the strategic incentive to export promotion as:

\[
SI^H = \frac{(n - 1)h'(x)h'(z)\Pi_2^H \Pi_{12}^H \Pi_{13}^H}{\Delta} \tag{8}
\]

where \( \Delta > 0 \) is the determinant of the equilibrium system. When \( \Pi_{13}^H > 0 \) this effect is positive under strategic substitutability \( (\Pi_{12} < 0) \) and negative under strategic complementarity \( (\Pi_{12} > 0) \): in the former case an increase in the

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9Given the symmetric equilibrium, I will drop the index \( i \) for the international firms and use the index \( H \) for the domestic one.
policy is going to reduce the strategies of the international firms and increase the profits of the domestic one, and in the latter case the opposite holds. All the results are inverted when $\Pi_{13} < 0$. It is now immediate to conclude with:

**Proposition 1.** When the number of firms is exogenous in the foreign market: a) if the export policy increases the marginal profitability of the domestic firm, there is (not) a strategic incentive to promote exports if strategic substitutability (complementarity) holds; b) if the export policy decreases the marginal profitability of the domestic firm, the opposite holds.

Notice that with just one domestic firm, the kind of policy does not depend on the number of international firms. The optimal policy implies an aggressive firm under strategic substitutability and an accommodating one under strategic complementarity. However, the result is sensitive to the number of domestic firms: if this is large enough, there is a bias against export promotion (Dixit, 1984, and Klette, 1994). In conclusion, the results on the optimal export policy are ambiguous when the market structures are exogenous.

### 2.2 Strategic policy with endogenous market structures

From the previous section we can infer that in standard models of competition in quantities and in prices, the foreign country gains from an increase in the number of international firms whenever this increases production or reduces the equilibrium prices. Therefore, it is interesting to investigate what happens when we allow for free entry in the foreign market.

We will assume that the number of potential entrants is great enough that a zero profit condition pins down the effective number of firms competing in the foreign market. The equilibrium conditions are the two first order conditions, (6) and (7), and the zero profit condition which binds on the international firms (since these do not profit from the optimal export policy):

$$\Pi [x, (n - 2)h(x) + h(z), 0] = F$$

(9)

Totally differentiating the system (6)-(7)-(9) we obtain a fundamental result for what follows (see the Appendix):

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As customary, we consider $n$ as a natural number in all the article (except when dealing with entry deterrence).
Proposition 2. Under free entry in the foreign market, a change in the export policy does not affect the equilibrium strategy of all the other firms but only their equilibrium number.

When the domestic policy is changed, the marginal profitability of the strategy of the domestic firm changes and its optimal strategy changes as well. Nevertheless, the policy change does not affect the marginal profitability for the other firms, and any impact on the market structure emerges through an impact on the number of competitors.\textsuperscript{11}

More specifically, notice that optimization by the foreign firms and the free entry condition constraining their number pin down both the strategy of each firm and the level of spillovers that each firm receives from the strategies of the other international firms and the domestic firm, namely both $x$ and $\beta$, which are therefore both independent from $s$. Since the domestic policy affects the strategy of the domestic firm but not the aggregate statistics $\beta = (n - 2)h(x) + h[z(s)]$, it follows that the number of firms must be influenced by the domestic policy. In particular, we have:

$$\frac{dn}{ds} = \frac{h'(z)\Pi_{13}^H/h(x)}{\Pi_{11}^H - h'(z)\Pi_{12}^H} \leq 0 \quad \text{if} \quad \Pi_{13}^H \geq 0$$

$$\frac{dz}{ds} = -\frac{\Pi_{13}^H}{\Pi_{11}^H - h'(z)\Pi_{12}^H} \geq 0 \quad \text{if} \quad \Pi_{13}^H \geq 0$$

A policy which makes the domestic firm more aggressive must reduce the number of international firms that can profitably be active in equilibrium, and vice versa.

In the initial stage, the government will choose the policy to maximize welfare. Using the envelope theorem and the previous results, we obtain the new strategic incentive to promote exports:

$$SI^H = \frac{h'(z)\Pi_{12}^H\Pi_{13}^H}{\Pi_{11}^H - h'(z)\Pi_{12}^H}$$

(10)

Its sign is simply the sign of $\Pi_{13}$, therefore we can conclude with:

Proposition 3. Under free entry in the foreign market, when the export policy increases (decreases) the marginal profitability of the domestic firm, there is (not) a strategic incentive to promote exports.

\textsuperscript{11}This result depends on the symmetric properties of the profit functions.
Notice that the result would not change in the presence of more than one domestic firm, as long as some entry of foreign firms takes place in equilibrium.\(^\text{12}\)

In conclusion, governments would always gain from unilateral commitments to implement export promoting policies that induce an aggressive behavior of the domestic firms that are active in global markets open to entry.\(^\text{13}\) Notice that the above analysis takes as given the policies of the other countries: later, we will present an equilibrium analysis in which all countries choose their policies independently.

In the rest of this section we will apply our general results to the theory of strategic trade policy. We will derive the optimal strategic unilateral trade policy in different models. The focus will be on specific subsidies, but similar results could be obtained with *ad valorem* subsidies.

### 2.2.1 Optimal export subsidy with Cournot competition

Consider the general model of quantity competition which allows for imperfect substitutability between goods and general cost functions. The gross profit of the domestic firm in the presence of a specific export subsidy is:

\[
\Pi^H = z [p(z, \beta_H) + s] - c(z)
\]

where we remember that \(z\) is the production of the domestic firm, \(p(\cdot)\) is the inverse demand depending on the spillovers from the production of other firms \(\beta_H\), \(c(\cdot)\) is the cost function and \(s\) is the subsidy. This profit function is clearly characterized by \(\Pi^H_{13} = 1 > 0\). The equilibrium first order conditions in the second stage where Nash competition takes place in the foreign market are:

\[
p(x, \beta) + xp_1(x, \beta) = c'(x)
\]

\[
s + p(z, \beta_H) + zp_1(z, \beta_H) = c'(z)
\]

\(^\text{12}\)Actually, it is immediate to verify that with \(n_H\) domestic firms, the equilibrium strategy of each firm would not change and the strategic incentive to promote exports would just be \(SI(n_H) = n_H SI(1)\). Under free entry there is not a terms of trade effect induced by an export promoting policy (which is present with entry barriers; see Dixit, 1984).

\(^\text{13}\)The result holds for markets in which a single domestic firm is active and subsidized. One should keep in mind that when other domestic firms are subsidized and endogenously enter in the market, entry would drive net domestic profits to zero. Venables (1985) studies a particular example of this case. See also Markusen and Venables (1988). Brander (1995) summarizes the results on entry for the reciprocal-markets model.
where $\beta = (n - 2)h(x) + h(z)$ is the spillover received by an international firm from the strategies of all the other firms in the market and $\beta_H \equiv (n - 1)h(x)$ is the spillover received by the domestic firm.

Let us now consider free entry. In the second stage we have also the zero profit condition:

$$xp(x, \beta) = c(x) + F$$

The equilibrium system expresses production levels and the number of firms as functions of the subsidy $s$, but we know from Proposition 2 that the production of foreign firms $x$ and their spillovers $\beta$ are actually unaffected by changes in the subsidy, while $z(s)$ and $\beta_H(s)$ depend on it. Consequently, we can write the welfare of the domestic country (2) as the profits of the domestic firm net of the tax revenue necessary to finance the subsidy:

$$W(s) = z(s)p(z(s), \beta_H(s)) - c[z(s)] - F$$  \hspace{1cm} (12)

whose maximization has an interior solution (without entry deterrence) if goods are imperfect substitutes or if marginal costs are increasing enough. If such an interior solution exists, it must satisfy the first order condition:

$$p(z(s), \beta_H) + z(s)[p_1(z(s), \beta_H) - p_2(z(s), \beta_H)h'(z)] = c'[z(s)]$$  \hspace{1cm} (13)

which is a complicated implicit expression. However, if we substitute this in the equilibrium first order condition for the domestic firm, we can derive the following expression for the optimal export subsidy:

$$s_H^* = -p_2(z, \beta_H)h'(z) \ z > 0$$  \hspace{1cm} (14)

It is interesting to derive the optimal subsidy for the case of homogenous goods: in such a case, an interior solution exists only if the marginal costs of production are increasing enough. When this is the case, the equilibrium price $p(X)$ is independent from the production of the domestic firm and from the subsidy because free entry for the international (not subsidized) firms determines total production (and the price) independently from both of them. Given this, the optimal subsidy simplifies to:

$$s_H^* = \frac{p}{\epsilon} > 0$$  \hspace{1cm} (15)
which is decreasing in the elasticity of demand (with respect to domestic production) \( \epsilon \equiv -p/zp' \).\(^{14}\) Notice that our expression for the optimal export subsidy is the exact opposite of the traditional neoclassical optimal trade policy for markets with homogenous goods. The latter requires an export tax inversely proportional to the elasticity of demand, so as to increase the price of exports compared to that of imports (i.e.: to improve the terms of trade). In our framework, an export subsidy of the same magnitude reduces the price of exports to conquer market shares in the foreign market and raise profits. In particular, the optimal policy implies that the domestic firm produces until its marginal cost equates the equilibrium price \( (p = c'(z)) \) and enjoys positive profits because returns to scale are decreasing at its production level.\(^{15}\) Nevertheless, when the elasticity of foreign demand increases, the optimal subsidy decreases. In the limit case of a perfectly elastic demand, which matches the case of a small open economy whose policy does not affect international equilibria, we reach the traditional outcome for which free trade is the optimal policy.

As an example, consider the case of a linear inverse demand \( p = a - X \), where \( X \) is total production, and a convex cost function that we assume to be quadratic for simplicity, with \( c(x) = x^2/2 \). Looking at the Cournot equilibrium between \( n \) firms for a given subsidy \( s \) for the domestic one, and imposing the free entry condition, we obtain the equilibrium production for each international firm:

\[
x = \sqrt{\frac{2F}{3}}
\]

and the number of firms:

\[
n = (a - s/2)\sqrt{3/2F} - 2
\]

which imply total production \( X = a - \sqrt{8F/3} \) and price \( p = \sqrt{8F/3} \). Consistently with Proposition 2, the subsidy does not affect the individual production of the other firms, but decreases their number. The equilibrium production of

\(^{14}\)Notice that \( p \) is independent from the production of the domestic firm and from the subsidy because free entry for the international (not subsidized) firms determines total production (and the price) independently from both of them.

\(^{15}\)Notice that the optimal subsidy would be the same in the presence of other domestic firms: there is not a terms of trade effect because the equilibrium price is independent from the subsidy, while domestic firms crowd out foreign firms.
the subsidized firm is instead \( z(s) = \sqrt{2F/3} + s/2 \), which generates net profits \( \pi_H = (3/8)(s + \sqrt{8F/3})^2 - F \). The government maximizes profits net of the tax revenue necessary to finance the subsidies:

\[
W(s) = z(s)\sqrt{\frac{8F}{3}} - \frac{z(s)^2}{2} - F
\]

(16)

that is maximized by:

\[
s_H^* = \sqrt{\frac{8F}{3}} > 0
\]

(17)

which implies that the domestic firm produces the double than any other international firm. Its net profits are \( \pi_H = 3F \) and domestic welfare is \( W = F/3 \).\(^{16}\)

We will return to this example in the next section.

When the welfare maximization has a corner solution, the optimal subsidy is high enough to deter entry of international firms. It is easy to verify that this outcome emerges in the relevant case of homogenous goods and constant marginal costs of production.\(^{17}\) Intuitively, the same outcome will occur for high levels of substitutability between products or/and the cost function is not increasing too much with the production level.

The prohibitive subsidy is the one that eliminates profits for any potential entrant. Formally, since we defined \( n \) as the total number of firms including the domestic one, the prohibitive subsidy must be (an epsilon larger than) the one which induces exactly zero profits for a single entrant, that is the one satisfying \( n = 2 \). Therefore, the prohibitive subsidy is implicitly given by the following condition:

\[
xp(x, z(s_H^*)) - c(x) = F
\]

(18)

The intuition for the optimality of the prohibitive subsidy is the following. Free entry pins down the equilibrium price level as long as some of the foreign

\(^{16}\)Notice that when the fixed cost of entry decreases, the level of concentration in the market is reduced and the optimal subsidy goes down: in the limit case of perfect competition (zero fixed costs) we obtain the traditional result for which free trade is optimal.

\(^{17}\)To verify this notice that for low values of the subsidy that allow entry of international firms, welfare (12) becomes \( W(s) = z(s)p(X) - cz(s) - F \), where the equilibrium price is independent from the subsidy and \( c \) is the constant marginal cost of production. Given this, welfare is always increasing in the subsidy (since \( p(X) > c \)) and it is optimal to set it high enough to deter entry.
firms enter. This implies that the choice of the subsidy does not affect the equi-
librium price at which the domestic firm will sell its production but increases
its market share. Since there are fixed costs of production, an increase in the
market share reduces average costs and therefore it increases net profits. Con-
sequently, it is optimal to raise the market share as much as possible, which
amounts to full entry deterrence.\footnote{For instance, let us consider the case of homogenous goods with a linear demand $p = a - X$
and constant marginal cost $c$. Imagining that there is entry in equilibrium and imposing the
free entry condition for a given subsidy $s$, we obtain the equilibrium production for each
international firm $x = \sqrt{F}$ and the number of firms $n = (a - c - s)/\sqrt{F} - 1$, which imply
total production $X = a - c - \sqrt{F}$. The equilibrium production of the subsidized firm is
$z(s) = \sqrt{F} + s$. The government maximizes profits net of the tax revenue necessary to finance
the subsidies:
$W(s) = \sqrt{F} \left( \sqrt{F} + s \right) - F$
Since this is always an increasing function of $s$, it is optimal to increase subsidization as long
as there is entry. But entry is deterred for any subsidy larger than:
$s_H^* = a - c - 3\sqrt{F} > 0$
which makes impossible for a single entrant to be active (with $n \geq 2$ the single entrant
obtains non-positive profits). This prohibitive subsidy is the optimal one and generates total
production $z = a - c - 2\sqrt{F}$, which is below the free trade level.}

Summing up:

**Proposition 4.** Under competition in quantities with free entry,
the optimal unilateral trade policy requires always a positive export
subsidy. With homogenous goods and increasing marginal costs, the
optimal subsidy is a fraction $1/\epsilon$ of the price, where $\epsilon$ is the elasticity
of the international demand.

### 2.2.2 Optimal export subsidy with Bertrand competition

Consider our general model of price competition with a (specific) export subsidy,
so that the gross profit function for the domestic firm is:

$$
\Pi^H = (p_H - c + s) D(p_H, \beta_H)
$$

where we remember that $D(\cdot)$ is the direct demand depending on the price of
the domestic firm $p_H$ and on the spillovers from the prices of the other firms

\footnote{For instance, let us consider the case of homogenous goods with a linear demand $p = a - X$
and constant marginal cost $c$. Imagining that there is entry in equilibrium and imposing the
free entry condition for a given subsidy $s$, we obtain the equilibrium production for each
international firm $x = \sqrt{F}$ and the number of firms $n = (a - c - s)/\sqrt{F} - 1$, which imply
total production $X = a - c - \sqrt{F}$. The equilibrium production of the subsidized firm is
$z(s) = \sqrt{F} + s$. The government maximizes profits net of the tax revenue necessary to finance
the subsidies:
$W(s) = \sqrt{F} \left( \sqrt{F} + s \right) - F$
Since this is always an increasing function of $s$, it is optimal to increase subsidization as long
as there is entry. But entry is deterred for any subsidy larger than:
$s_H^* = a - c - 3\sqrt{F} > 0$
which makes impossible for a single entrant to be active (with $n \geq 2$ the single entrant
obtains non-positive profits). This prohibitive subsidy is the optimal one and generates total
production $z = a - c - 2\sqrt{F}$, which is below the free trade level.}
This profit function clearly satisfies $\Pi_H^* = -p_H^2 D_1 > 0$.

As pointed out first by Eaton and Grossman (1986), the optimal trade policy under barriers to entry requires an export tax. Here, however, we will focus on the case of free entry, in which the equilibrium conditions in the second stage and the zero profit condition are:

\begin{align*}
(p - c)D_1(p, \beta) + D(p, \beta) &= 0 \\
(p_H - c + s)D_1(p_H, \beta_H) + D(p_H, \beta_H) &= 0 \\
(p - c)D(p, \beta) &= F
\end{align*}

where $\beta = (n - 2)g(p) + g(p_H)$ is the spillover received by an international firm from the strategies of all the other firms in the market and $\beta_H = (n - 1)g(p)$ is the spillover for the domestic firm. This system expresses prices and the number of firms as functions of the subsidy $s$, but we know from Proposition 2 that the price of foreign firms $p$ and their spillovers $\beta$ are independent from the subsidy, while $p_H(s)$ and $\beta_H(s)$ depend on it. Therefore, assuming that the cost of the subsidy is simply given by the tax revenue necessary to finance it, we can write the welfare of the domestic country (2) as:

\begin{equation}
W(s) = [p_H(s) - c] D [p_H(s), \beta_H(s)] - F = [p_H(s) - c] D [p_H(s), \beta + g(p) - g(p_H)] - F \quad (20)
\end{equation}

which is maximized by a subsidy satisfying the first order condition:

\begin{equation}
D(p_H, \beta_H) + (p_H - c) [D_1(p_H, \beta_H) - D_2(p_H, \beta_H) g'(p_H)] = 0 \quad (21)
\end{equation}

If we now substitute this in the equilibrium first order condition for the domestic firm, we can derive the following expression for the optimal export subsidy:

\begin{equation}
s_H^* = \frac{(p_H - c) D_2(p_H, \beta_H) g'(p_H)}{-D_1(p_H, \beta_H)} > 0 \quad (22)
\end{equation}

Also this is an implicit expression, since on the right hand side $p_H$ depends on the optimal subsidy, however this expression makes clear our main point: the optimal export subsidy must be positive.

Summarizing, under price competition and free entry, an export subsidy is always optimal, since it helps the domestic firm to lower its price in the foreign market. The result overturns common wisdom for models with strategic
complementarity and barriers to entry. An accommodating behavior is not anymore optimal because it would just induce new firms to enter. The only chance for the government to increase the profits of the domestic firm is to induce an aggressive behavior: the domestic firm undercuts its competitors, gains market shares and spreads a low mark up over a large portion of the market, leaving the few remaining firms with zero profits. This outcome can only be reached with an export subsidy. Summing up:

**Proposition 5.** Under competition in prices with free entry, the optimal unilateral trade policy requires always a positive export subsidy.

An explicit characterization can be obtained in the case of a Logit demand,

\[
D_i = \frac{Ye^{-\xi p_i}}{\sum_{j=1}^{n} e^{-\xi p_j}},
\]

with \(Y > 0\) representing total demand in the sector, and with \(\xi > 0\). In this case, international firms choose the price \(p = c + F/Y + 1/\xi\) and it is easy to derive that the optimal subsidy must induce a price for the domestic firm equal to \(p_H(s^*_H) = c + 1/\xi\), which requires an optimal export subsidy equal to:

\[
\frac{F}{Y} > 0
\]

Notice that when the size of the fixed costs relative to the size of the market decreases, the endogenous level of concentration in the market is reduced, and the optimal subsidy is lower.

Another explicit result for the optimal export subsidy can be derived in models with isoelastic demand which can be microfounded in a standard way and are widely used in international trade theory. Consider a Dixit and Stiglitz (1977) demand function as:

\[
D_i = \frac{Y p_i^{-\frac{1}{\alpha}}}{(1 + \alpha) \sum_{j=1}^{n} p_j^{-\frac{1}{\alpha}}}
\]

\[\text{with } \theta \in (0, 1) \text{ and } \alpha > 0, \text{ to be maximized under the budget constraint } C_0 + \sum_{j=1}^{n} p_j C_j = Y, \text{ where } C_0 \text{ is the numeraire.}\]
In this case the optimal export subsidy, derived in the Appendix, determines an equilibrium price \( p_H(s_H^*) = c/\theta \) for the domestic firm which is lower than the equilibrium price of the other international firms \( p = cY/\theta[Y - F(1 + \alpha)] \). However, one can verify that the reduction in the number of these international firms maintains the price index at the same level as under free trade.

3 APPLICATIONS

In this section we adopt and extend our analysis for new applications on strategic policy for international markets whose structure is endogenous. Beyond subsidization, many other policies can affect the profits of exporting firms: for instance, policies which increase demand for the domestic product, reduce transport costs for exporting firms or promote R&D (Spencer and Brander, 1983). In the first subsection we evaluate the incentives to adopt unilaterally R&D subsidies which provide a strategic advantage for domestic firms participating to the competition for international markets. In the following subsection we evaluate the strategic incentives of the monetary authority of a country to intervene unilaterally on a fixed exchange rate to support domestic firms active abroad. Finally, we develop an equilibrium analysis for the case in which multiple countries can simultaneously choose their strategic trade policy (for simplicity this analysis is limited to a particular case, that of competition in quantities): our results confirm that the strategic incentives to subsidize exports persist also in a Nash equilibrium between multiple countries.

3.1 R&D Policy

In this section we briefly address the role of R&D policy in supporting domestic firms active abroad. R&D policy is quite relevant for high-tech industries: its main aspect involves R&D subsidies, but there are other forms of R&D promotion as public investment in complementary R&D or the strengthening of the protection of intellectual property rights for the domestic firms. One can analyze the role of unilateral R&D policy focusing on the competition for international markets rather than the competition in international markets. Traditional models of patent races are nested in our general framework and can be used to study
R&D policy for firms investing in some forms of innovation to conquer foreign markets. For instance, consider a standard international patent race where each firm $i$ invests a flow of investment $x_i$ in the continuous time. This investment delivers innovations according to a standard Poisson stochastic process characterized by an instantaneous arrival rate of innovations $h(x_i)$, which is a positive and concave function. When one of the firms innovates, it obtains a rent $V$ and the race is over. The R&D subsidy $s_i$ is assumed to be proportional to the investment flow. Given a constant interest rate $r$, the expected profit function for firm $i$ can be expressed as:

$$\Pi(x_i, \beta_i, s_i) = \frac{h(x_i)V - x_i(1 - s_i)}{r + h(x_i) + \beta_i}$$

(24)

which is clearly nested in our general functional form (1). Notice that $\Pi_{12}^H > 0$ and $\Pi_{13}^H > 0$, therefore in case of a fixed number of international firms (Proposition 1), it would be optimal to tax domestic R&D (to slow down the aggregate investment rate), while under the assumption of endogenous entry in the international competition for the market (Proposition 3) a positive R&D subsidy is always optimal. Adopting the usual procedure, it is easy to verify that the optimal unilateral R&D subsidy satisfies:

$$s_H^* = \frac{1}{1 + \frac{V(r+\beta_H)}{h(z)V - z}} \in (0,1)$$

(25)

where the investments of the domestic firm, $z$, and of the foreign firms, $x$, satisfy $h'(z)V = h'(x)(V - F) = 1$. Once again, the subsidy allows the domestic firm to commit to a more aggressive strategy, which is now represented by a larger investment flow. Summarizing:

**Proposition 6. In a patent race between international firms, a) when the number of firms is exogenous it is optimal to set a R&D tax, but b) when entry of international firms is free the optimal unilateral R&D policy requires always to set a positive R&D subsidy.**

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20 A generalization of the optimal R&D subsidy within a general equilibrium model of endogenous growth can be found in Etro (2008,b).
3.2 Exchange rate policy

An important application which deserves more attention is to competitive devaluations adopted with the specific aim of supporting exports. Economic theory is ambiguous on their merits. The traditional Mundell-Fleming model emphasizes the beggar-thy-neighbour effects of unilateral devaluations. However, the recent new open-economy macroeconomics shows that these devaluations can be beggar-thy-self policies.\(^{21}\) In front of this theoretical ambiguity it is difficult to make sense of the common wisdom according to which unilateral devaluations provide a positive strategic advantage on the international markets. In this section we evaluate the strategic incentives to exchange rate devaluations in a model based on Dornbusch (1987), where the incidence of exchange rate variations on prices is endogenous.

The effects of exchange rate policy for exporting firms crucially depend on the location of production, on whether local currency pricing or producer currency pricing holds\(^{22}\) and on the strategic reaction of firms to the policy. In our partial equilibrium context, we will focus on the strategic effects of a devaluation on the domestic firm. Clearly, a devaluation has other consequences in general equilibrium, but the point here is just to understand whether the usual claim that a devaluation gives a strategic advantage to exporting firms is correct. Our focus will be on a particular situation where all firms produce in their domestic country, bear production costs in domestic currency, choose their strategy taking into account the exchange rate and then export abroad. Under price competition this corresponds to the case of producer currency pricing. Such a case is typical of medium and small firms which are active at a national level, often producing typical domestic products and exporting some of them abroad, but also of larger firms which are not directly active in the foreign market under consideration but sell their goods to distributors in that market.\(^{23}\) We will study separately the cases of quantity competition and price competition. The bottom line will be that competitive devaluations are always desirable to provide a strategic

\(^{22}\)See Engel (2000) and Betts and Devereux (2000).
\(^{23}\)The alternative situation, which is not relevant for our purposes, emerges when international firms produce and compete abroad with independent production units. This is typical of multinational firms which are directly active in other countries where they sell their products.
advantage to domestic firms when entry in the foreign markets is free.\footnote{Potentially, one could extend this framework to derive an optimal competitive devaluation comparing its benefits on the export side with its costs on the import side. However, this remains a partial equilibrium analysis. One should always keep in mind that in general equilibrium and in the absence of pervasive market imperfections, purchasing power parity holds and it requires automatic adjustments of nominal variables - which undermine the effectiveness of exchange rate policy.}

### 3.2.1 Competitive devaluations with Cournot competition

Imagine a foreign market with competition in quantities. Foreign demand for good \( i \) is as usual \( p_i = p(x_i, \beta_i) \) but revenues in domestic currency are \( E_i x_i p_i \) where \( E_i \) is the price of the foreign currency in terms of currency of country \( i \), that is the exchange rate of this country. For expository purposes, imagine an initial situation where, without loss of generality, all the exchange rates (with the foreign country where firms compete) are unitary. If the domestic country can adopt a competitive devaluation and rise the exchange rate to the level \( E \), the profit of the domestic firm becomes:

\[
\Pi^H = E z p(z, \beta_H) - c(z) \tag{26}
\]

which can be rewritten in our framework as \( \Pi^H(z, \beta_H, s) \) where \( s = E - 1 \), implying \( \Pi^H_{13} = p + z p_1 = c'(z)/E > 0 \). Hence, our general results apply and tell us that after a devaluation the domestic firm will increase its production level. Under barriers to entry, as long as strategic substitutability holds, the other firms will decrease production so that the market share of the domestic firm increases (as it was shown by Dornbush, 1987): this creates a strategic incentive to devaluate. Also under free entry the domestic firm expands its market share, but the other firms produce the same as before the devaluation, and some of them exit from the market. Applying Propositions 1 and 3, we have:

**Proposition 7.** Under quantity competition, a) when the number of firms is exogenous there is a strategic incentive for competitive devaluations if strategic substitutability holds and b) when entry is free there is always a strategic incentive for competitive devaluations.

Notice that a devaluation always increases domestic production and exports.
3.2.2 Competitive devaluations with Bertrand competition

The case of price competition is the most interesting, since it is the usual case under study in macroeconomic models and probably the most realistic for our purposes.

Imagine again an initial situation where all the exchange rates are unitary including the price of the foreign currency in terms of domestic currency, \( E \). Notice that, if \( p_H^* \) is the price of the domestic good in foreign currency, the price of the same good in domestic currency is \( p_H = E p_H^* \). If the latter is constant, a devaluation (an increase in \( E \)) will reduce the price in foreign currency, and an appreciation of the exchange rate will increase it. However, prices in domestic currency for foreign segmented markets can be changed after a devaluation and our purpose is exactly to check how they are changed.

Since production takes place at home and demand depends on prices in foreign currency, the relevant profit function for the domestic firm is:

\[
\Pi^H = (p_H - c) D \left[ \frac{p_H}{E}, \sum g(p_j^*) \right] = (E p_H^* - c) D (p_H^*, \beta_H) \tag{27}
\]

which can be rewritten in our framework with \( z = 1/p_H^* \) and \( s = E - 1 \). With such a change of variables, the strategic variable for each firm becomes the price in foreign currency. Clearly, for all the international firms except the domestic one, the price is the same in foreign and domestic currency, \( p_j^* = p_j \) for \( j \neq H \).

As usual, the incentives to change strategy for the domestic firm depend on the cross effect \( \Pi_{13}^H = -p_H^* D + p_H^* D_1 \), which is positive in equilibrium. Therefore, the price of the domestic firm in foreign currency \( p_H^* \) is always decreasing in the exchange rate, that is after a devaluation. In general, Proposition 1 implies that a competitive devaluation is not strategically desirable under barriers to entry. Such a policy forces the domestic firm to decrease its price in foreign currency, which induces also the other firms to do the same, reducing profits for all the firms. Actually, there is a strategic incentive to appreciate the currency, which induces the domestic firm to increase its own price in foreign currency and the other firms to do the same.\(^{25}\)

\(^{25}\)Of course, this is just the strategic incentive for the government: an appreciation would also have a negative direct effect on profits, reducing the mark-up of the domestic firm, and finally, it will induce other effects for domestic welfare like a reduction in the price of imports.
When entry is free, the domestic firm does not obtain a strategic advantage when induced to increase its own price because this would promote entry in the foreign market. According to Proposition 3, there is a strategic incentive to devaluate the exchange rate. This would reduce the price of the domestic firm in the foreign currency. Foreign firms would not change their own prices, but fewer would enter in the market so that the market share of the domestic firm would expand - in this case, a devaluation has also a direct beneficial effect, since it increases revenues of the domestic firm in domestic currency.\footnote{The positive direct and strategic effects of a devaluation should be compared with the costs in terms of a higher price of imports, which is beyond the scope of this discussion.} Summing up, the usual claim that devaluations give a strategic advantage to exporting firms is correct only for foreign markets whose structure is endogenous:

**Proposition 8.** Under price competition, a) when the number of firms is exogenous, there is a strategic incentive to appreciate the domestic currency, but b) when entry is free there is a strategic incentive for competitive devaluations.

The bottom line is quite intuitive. Devaluations can be deleterious for exporting firms when they induce a war between international firms to reduce prices in foreign currency and this happens when there are clear barriers to entry. However, when entry is free, international firms cannot undertake such a war and the domestic firm can unilaterally decrease its price in foreign currency expanding its market share: only in this case there is a strategic incentive toward competitive devaluations.

3.3 Equilibrium trade policy

In this section we provide an equilibrium analysis for the case in which multiple countries choose their export promotion policies.\footnote{I am extremely thankful to a referee for pointing out this case and leading to its characterization.} To appreciate the importance of this analysis, consider first the traditional case where there are two countries with two firms active in a third market, and both countries independently choose an export subsidy. This situation, studied first by Brander and Spencer (1985) in a model of competition in quantities with strategic substitutability, generates an inefficient symmetric Nash equilibrium in which both
countries engage in excessive subsidization of their exports: although export subsidies are unilaterally optimal, they are jointly suboptimal (for the countries involved) when one considers equilibrium behavior. Analogously, in case of strategic complementarity, the Nash equilibrium is characterized by suboptimal export taxation by both countries. Of course, these results extend to a larger number of countries and firms, but, as we will see in this section, they depend again on the exogeneity of the market structure.

Let us consider the general case where each one of $m$ countries can subsidize or tax the exports of a single national firm to an international market, but the number of firms that are ultimately active in the market is endogenous. The timing of the game is the following:

1) $m$ countries independently choose their export subsidies (taxes if negative) $s = [s_1, s_2, ..., s_m]$ maximize their welfare functions $W_i$;
2) simultaneous entry of $n$ firms occurs endogenously;
3) all the $n$ active firms independently choose their strategies $x_i$ to maximize their profits $\pi_i$.

We will provide a constructive approach to the equilibrium analysis focusing on an example of competition in quantities with homogenous goods where the demand is linear, $p = a - X$, and the cost function is quadratic, $c(x) = x^2/2$ (already used in Section 2.2.1). The main intuitions extend to the general case (as shown in the Appendix) and to other models.

Solving for the subgame perfect equilibrium in pure strategies by backward induction we will show that the Nash equilibrium trade policy is characterized by a limited and endogenous number of countries adopting the same unilaterally optimal export subsidy, and by the other countries committing to free trade.

Let us consider stage 3) first. The set of subsidies is given and, without loss of generality, we order the countries by decreasing subsidies: $s_1 \geq s_2 \geq ... \geq s_m$. At this stage also the number of active firms $n$ is known. The Cournot

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28The same happens under competition in prices. As well known, also competitive devaluations lead to inefficient equilibrium behavior.

29We assume that the number of countries (and potential firms) is high enough that $n < m$ in equilibrium. Otherwise, the game would revert to one with an exogenous number of firms. For simplicity, we also assume that when a country cannot induce entry of its firm and cannot improve welfare by means of an active policy (a subsidy or a tax), the country commits to free trade.
equilibrium implies production \( x_i = (a - X + s_i)/2 \) and profits \( \pi_i = 3x_i^2/2 - F \) for each one of them. Summing up to obtain total production \( X(n, s) = \left( a(n + \sum_{j=1}^{n} s_j) \right)/(2 + n) \), we notice that this is increasing in the number of active firms and in their subsidy. This allows us to express the production level of each firm as:

\[
x_i(n, s) = \frac{a}{2 + n} + \frac{1}{2} \left( s_i - \frac{\sum_{j=1}^{n} s_j}{2 + n} \right)
\]  

(28)

Let us move to stage 2). Since the profits of the active firms are decreasing in the subsidies, there must be a marginal firm obtaining zero profits. As usual in this literature, we are neglecting the integer constraint on the number of firms: this is a good approximation as long as the equilibrium number of firms is large enough (i.e.: \( a \) is large enough). The free entry condition determining \( n \) is \( \pi_n = 0 \), and requires that the production of the marginal firm is given by \( x_n = \sqrt{2F/3} \) and the total production is \( X(s_n) = a + s_n - \sqrt{8F/3} \), which depends positively on the critical subsidy \( s_n \).\(^30\) This generates an equilibrium price depending on the crucial subsidy as follows:

\[
p(X(s_n)) = \sqrt{\frac{8F}{3}} - s_n
\]  

(29)

The equilibrium production for each active firm can be derived as follows:

\[
x_i = \sqrt{\frac{2F}{3}} + \frac{s_i - s_n}{2}
\]  

(30)

Summarizing, given any set of national subsidies, the most subsidized firms must be active in the market, with the marginal firm producing enough to break even and the other firms producing a quantity that is increasing in their subsidy. The number of active firms depends on all the subsidies according to the following relation:

\[
n = \frac{2a - \sum_{j=1}^{n} s_j}{\sqrt{\frac{8F}{3}} - s_n} - 2
\]

Finally, let us move to stage 1). To characterize the subgame perfect equilibrium we need to find a set of subsidies such that each one maximizes the welfare

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\(^{30}\)This relies on the assumption that the equilibrium number of firms is a natural number. If this was not the case, the equilibrium number should be the smallest integer \( \bar{n} \) satisfying \( x_n(\bar{n}, s) \geq \sqrt{2F/3} \) and \( x_{n+1}(\bar{n} + 1, s) < \sqrt{2F/3} \). Of course, this number would depend on the full set of subsidies.
of the corresponding country (domestic profits net of the cost of the subsidy) taking as given the other subsidies and the equilibrium of the subgames.\footnote{The characterization of the equilibrium below relies on the assumption that the equilibrium number of firms is a natural number. If this was not the case, the integer number of firms in equilibrium $n_e$ would depend on the subsidy of countries $1, 2, \ldots, n_{e-1}, n_{e-1} + 1$, and each active country would choose its subsidy to maximize:}

We first establish an equilibrium requirement for the countries $i < n$ that are defined by construction as the countries with active firms obtaining positive profits. Using (29) and (30), the welfare $W_i$ of a country $i < n$ can be expressed as:

$$W(s_i, s_n) = \left( \sqrt{\frac{2F}{3} + \frac{s_i - s_n}{2}} \right) \left( \sqrt{\frac{8F}{3} - s_n} \right) - \frac{1}{8} \left( s_i - s_n + \sqrt{\frac{8F}{3}} \right)^2 - F$$

Its maximization for $s_i$ taking as given $s_n$ delivers:

$$s^*(s_n) = \sqrt{\frac{8F}{3} - s_n}$$

which depends negatively on the critical subsidy.\footnote{Notice that $s^*(s_n) \geq s_n$, as required by construction, if $s_n \leq s^*(s_n)/2$.} In equilibrium, each country $i < n$ must adopt this subsidy.

We now claim that the equilibrium must imply $s_i = 0$ for all the countries $i \geq n$, and therefore that there are no profitable deviations from free trade for all of these countries.

Consider the marginal country $n$, which is defined by construction as the country whose subsidy $s_n \in (s_{n+1}, s_{n-1})$ leads its firm to break even in the last stage. This country could only avoid this outcome with a unilateral deviation as $\hat{s} \geq s_{n-1}$ or $\hat{s} < s_{n+1}$.\footnote{This depends again on the assumption that $n$ is a natural number such that the marginal firm breaks even. As a consequence of this, a deviation given by a small increase in the subsidy to $\hat{s} \in (0, s_{n-1})$ does not change the equilibrium strategy and the equilibrium (zero) profits of the national firm, but simply increases the total output and reduces the price. Accordingly, the deviation does not increase net profits but has a welfare cost due to the cost of the subsidy.} We now show that this country cannot gain from

$$W_i(s) = 3x_i(n_s, s)^2/2 - F - s_i x_i(n_s, s)$$

taking as given the other subsidies. Closed form solutions for the equilibrium subsidies are not available. The approximation in the text, that considers $n$ as a natural number, allows us to derive explicitly the equilibrium subsidies, number of firms and strategies.

26
both kinds of deviation. First, a deviation with a positive and large subsidy $\hat{s} \geq s_{n-1} = s^*(0)$ would turn the other subsidized firms (of countries $i < n$) into the marginal firms, with $s_n = s^*(0) = \sqrt{8F/3}$. However, from (31) it emerges that $W(\hat{s}, s^*(0)) \leq 0$ for any deviation $\hat{s}$, therefore such a deviation from the equilibrium strategy cannot be profitable. Second, any unilateral deviation with a negative subsidy $\hat{s} < s_{n+1} = 0$ would lead to the exit of the national firm (in favor of another unsubsidized firm) without inducing any welfare gain compared to the equilibrium strategy.

Consider now the other countries $i > n$. In the proposed equilibrium they choose free trade, but their firms do not enter in the international market. These countries cannot gain from unilateral deviations for analogous reasons to those of the marginal country: a positive subsidy inducing entry of the national firm would reduce welfare, and a negative one would not change the outcome of the game.

In conclusion, the equilibrium generates the same optimal unilateral subsidy found in (17) for an endogenous number of countries and free trade for the others:

$$s^*_H = \sqrt{\frac{8F}{3}} \quad \text{for } i < n, \quad s^*_H = 0 \quad \text{for } i \geq n$$

where $n$ is given by:

$$n = \frac{1}{2} \left( a\sqrt{\frac{3}{2F} - 1} \right)$$

In this example, all firms receiving a positive subsidy produce the double of the marginal firm and obtain positive net profits $\pi_i = 3F$ for $i = 1, 2, \ldots, n - 1$, but the number of countries able to exploit the advantages of strategic export subsidization is limited by the size of the market. In our example, each one of these countries obtains a welfare gain $W = F/3$ relative to free trade - but notice that the equilibrium price remains at the free trade level $p = \sqrt{8F/3}$ (which shows a Pareto improvement of the allocation of resources).

In general, the welfare gain is identical for all the countries that actively subsidize their firms, and the same as in the case of a unilateral optimal policy - indeed, even through coordination those countries could not reach a better outcome. In other words, in the presence of endogenous market structures, strategic trade policy is not a beggar-thy-neighbour policy in the traditional
sense. Nevertheless, only a limited number of countries can exploit the benefits of this policy: the adoption of export subsidies by some countries induces the exit of international firms compared to the free trade equilibrium.

The result can be easily extended to general demand and cost functions (see the Appendix). It is important to notice that nothing of the argument above relies on strategic substitutability or complementarity. The optimal unilateral export subsidies would emerge in the Nash equilibrium with endogenous entry also under strategic complementarity (contrary to the case of an exogenous number of firms, which would give raise to export taxes in equilibrium). For the same reason, the result applies in the presence of competition in prices: in equilibrium a limited number of countries adopts the same positive subsidies derived in the previous sections as optimal unilateral policies, while the marginal firm active in the market is not subsidized. The important point is that the traditional conclusion for which export promotion is unilaterally optimal but jointly suboptimal does not appear to be robust in the presence of endogenous market structures.

4 CONCLUSIONS

In this article we adopted a simple model to show the general optimality of unilateral export promotion policies in foreign markets whose structure is endogenous. The theoretical implications are particularly strong for markets with competition in prices and it is worthwhile to summarize them: the opening up of such markets to free entry of foreign firms would change the optimal unilateral trade policy for the exporting countries from taxation to subsidization of the exports and would create new strategic incentives to implement other forms of strategic export promotion as competitive devaluations.

Our analysis of another interesting case, that of competition in quantities with homogenous goods, has shown that, in the absence of prohibitive subsidies, the optimal export subsidies create profits for the corresponding country without affecting the equilibrium price in the international market. Therefore, they improve the allocation of resources compared to the free trade outcome, a result that holds also when other countries can choose their policies as well. A possible policy implication is that banishing export subsidies, one of the principles of the
WTO, may not be a good idea, at least for imperfectly competitive markets. Moreover, similar limitations could simply push countries toward the adoption of other indirect forms of export promotion as investments in infrastructures or R&D promotion that provide a competitive advantage for domestic firms in the international markets.

Our model could be relevant for trade between developed and developing countries whose markets open up. A spectacular example is given by China and India, whose huge markets are starting to massively import from the Western world. Our results suggest that the gains for the Western world from promoting exports in concentrated markets could be quite large.

Further theoretical research could extend these results. On one side, one could study more complex models of interaction between firms and governments and introduce this set up in a standard two country framework of international trade. Moreover it would be interesting to extend the model of strategic trade policy for the domestic market in presence of free entry. On the other side, one could analyze the strategic effects of devaluations on both foreign and domestic markets. Finally, the welfare and equilibrium analysis could be extended to more general frameworks.

**APPENDIX**

*Proof of Proposition 1.* Let us totally differentiate the system (6) and (7) under the stability assumption:

$$\Delta \equiv \Pi_{11}^H \Pi_{11} + (n - 2)\Pi_{12}^H h'(x) - (n - 1)\Pi_{12} h'(x) h'(z) > 0$$

and:

$$\Pi_{11}^H + \Pi_{11} + (n - 2)\Pi_{12} h'(x) < 0$$

where $\Delta > 0$ is the determinant of the equilibrium system. Moreover let us assume:

$$\Pi_{11} + (n - 2)\Pi_{12} h'(x) < 0$$

which always holds under strategic substitutability, and under strategic complementarity if the number of firms is small enough. The equilibrium strategies $x = x(s)$

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34See Etro (2009) for a preliminary investigation of these topics.
and $z = z(s)$ are two functions of the domestic policy $s$ with:

$$\frac{dx(s)}{ds} = \frac{\Pi_{12} \Pi_{13}^H h'(z)}{\Delta} \leq 0 \quad \text{if} \quad \Pi_{12} \Pi_{13}^H \leq 0$$

$$\frac{dz(s)}{ds} = -\frac{[\Pi_{11} + (n-2)\Pi_{12} h'(x)] \Pi_{13}^H}{\Delta} \geq 0 \quad \text{if} \quad \Pi_{13}^H \geq 0$$

In the initial stage the government will choose the policy to maximize welfare. Using the envelope theorem and the previous results, we obtain the strategic incentive to export promotion as:

$$SI = \frac{(n-1)h'(x)h'(z)\Pi_{12}^H \Pi_{13}^H}{\Delta}$$

whose sign proves the proposition.

Proof of Proposition 2. To verify the comparative statics of the system (6)-(7)-(9) with respect to $s$, let us use the definitions $\beta = (n-2)h(x) + h(z)$ and $\beta_H \equiv (n-1)h(x)$ to rewrite it in terms of the three unknown variables $x$, $z$ and $\beta_H$:

$$\Pi_1 [x, h(z) - h(x) + \beta_H, 0] = 0$$

$$\Pi_H^H [z, \beta_H, s] = 0$$

$$\Pi [x, h(z) - h(x) + \beta_H, 0] = F$$

The second equation provides an implicit relationship $z = z(\beta_H, s)$ with $\partial z / \partial \beta_H = -\Pi_{12}^H / \Pi_{11}^H$ and $\partial z / \partial s = -\Pi_{13}^H / \Pi_{11}^H > 0$. Substituting this expression we obtain a system of two equations in two unknowns, $x$ and $\beta_H$:

$$\Pi_1 [x, h(z(\beta_H, s)) - h(x) + \beta_H, 0] = 0$$

$$\Pi [x, h(z(\beta_H, s)) - h(x) + \beta_H, 0] = F$$

Totally differentiating the system, it follows that $x = x(s)$, $\beta_H = \beta_H(s)$ and $z = z(\beta_H(s), s)$ are the equilibrium functions with the following comparative statics:

$$\frac{dx}{ds} = 0 \quad \frac{d\beta_H}{ds} = h'(z) \frac{\Pi_{13}^H}{\Pi_{11}^H - h'(z) \Pi_{12}^H} \leq 0 \quad \text{iff} \quad \Pi_{13}^H \leq 0$$

$$\frac{dz}{ds} = -\frac{\Pi_{13}^H}{\Pi_{11}^H - h'(z) \Pi_{12}^H} \geq 0 \quad \text{iff} \quad \Pi_{13}^H \geq 0$$
This implies that the policy $s$ does not affect the strategy of the foreign firms $x$. Moreover, since $\beta_H \equiv (n - 1)h(x)$ we have $n = 1 + \beta_H/h(x)$ and

$$\frac{dn}{ds} = \frac{d\beta_H}{ds} h(x)^{-1} \leq 0 \text{ iff } \Pi_{13}^H \leq 0$$

which concludes the proof.

**Optimal export subsidy under price competition.** We solve for the optimal trade policy in a model of price competition with a demand function a la Dixit and Stiglitz (1977). To re-express the model presented in the text in terms of the variables of our general framework, let us set $x_i \equiv 1/p_i$ and $h(x_i) = x_i^{\frac{\theta}{\alpha}}$ so that, in the presence of a specific subsidy we have:

$$\Pi(x_i, \beta_i, s_i) = \frac{x_i^{\frac{\theta}{\alpha}} - (c - s_i)x_i^{\frac{1}{\alpha}}}{(1 + \alpha)[h(x_i) + \beta_i]} Y$$  \hspace{1cm} (33)

It follows that $\Pi_{12} > 0$ at the optimal point satisfying $\Pi_1 = 0$, which implies strategic complementarity, and $\Pi_{13} > 0$.

Let us solve for the optimal export subsidy under price competition and free entry. The price of the foreign firms $p$ and of the domestic firm $p_H$, and the number of firms $n$ solve a system of the equilibrium conditions:

$$\left[ p_H - \frac{c - s}{\theta} \right] \left[ (n - 1)p^{-\frac{\theta}{\alpha\sigma}} + p_H^{-\frac{\theta}{\alpha\sigma}} \right] = \left[ p_H^{\frac{1}{\alpha\sigma}} - (c - s)p_H^{-\frac{\theta}{\alpha\sigma}} \right]$$  \hspace{1cm} (34)

$$\left[ p - \frac{c}{\theta} \right] \left[ (n - 1)p^{-\frac{\theta}{\alpha\sigma}} + p_H^{-\frac{\theta}{\alpha\sigma}} \right] = \left[ p^{\frac{1}{\alpha\sigma}} - cp^{-\frac{\theta}{\alpha\sigma}} \right]$$  \hspace{1cm} (35)

and the free entry condition:

$$\frac{Y \left( p^{-\frac{\theta}{\alpha\sigma}} - cp^{-\frac{\theta}{\alpha\sigma}} \right)}{(1 + \alpha) \left[ (n - 1)p^{-\frac{\theta}{\alpha\sigma}} + p_H^{-\frac{\theta}{\alpha\sigma}} \right]} = F$$  \hspace{1cm} (36)

From (35) and (36) one can derive the price of the international firms as:

$$p = \frac{cY}{\theta[Y - F(1 + \alpha)]}$$

which is independent of $s$. The optimal subsidy maximizes:

$$W(s) = \frac{p_H^{-\frac{1}{\alpha\sigma}} (p_H - c)}{\left[ (n - 1)p^{-\frac{\theta}{\alpha\sigma}} + p_H^{-\frac{\theta}{\alpha\sigma}} \right]} - F = \frac{p_H^{-\frac{1}{\alpha\sigma}} (p_H - c) F(1 + \alpha)}{\left( p^{-\frac{\theta}{\alpha\sigma}} - cp^{-\frac{\theta}{\alpha\sigma}} \right) Y} - F$$
where we used (36) in the second line. It is immediate to verify that the optimal subsidy must satisfy the first order condition \( p_H = c/\theta \). Substituting for \( p_H \) in the equilibrium condition (34) one obtains the optimal subsidy:

\[
s^*_H = \frac{c(1-\theta)}{\theta \left\{ \frac{Y-F(1+\alpha)}{Y} \right\}^{\frac{1}{1+\alpha}} \left\{ \frac{Y(1-\theta)}{F(1+\alpha)} + \theta \right\} - 1} > 0
\]

which is increasing in \( F/Y \), the ratio between the fixed costs of production and the size of the market demand.

A generalization of the equilibrium analysis. Consider the three stage equilibrium model with a general demand function \( p = p(X) \) and a general cost function \( c(x) \) assumed convex enough to exclude an equilibrium with entry deterrence. We solve for the subgame perfect equilibrium by backward induction to prove the following result: under quantity competition with homogenous goods and increasing marginal costs, the Nash equilibrium trade policy is characterized by a limited number of countries adopting the same unilaterally optimal export subsidy, and by the other countries committing to free trade.

In stage 3), the set of subsidies by all the countries and the number of active firms are given. We order the countries by decreasing subsidies: \( s_1 \geq s_2 \geq \ldots \geq s_{n-1} \geq s_n \geq \ldots \geq s_m \). Each active firm \( i \) maximizes net profits:

\[
\pi_i = x_i \left[ p(X) + s_i \right] - c(x_i) - F
\]

where \( X \) is total production. The first order condition is:

\[
s_i + p(X) + x_i p'(X) = c'(x_i)
\]

which displays a positive relation between the subsidy of country \( i \) and the production of its firm \( x_i \) for a given total output. As a consequence, the profit of the firms must be increasing in the subsidy.\(^{35}\) Total output is of course increasing in the number of firms and in their subsidies.

Let us move to stage 2). Firms enter in the market as long as they can foresee non-negative profits. This implies that the most subsidized firms enter, that the

\[
\frac{d\pi_i}{ds_i} = \frac{dx_i}{ds_i} \left[ -x_ip'(X) \right] + x_is_i > 0
\]

therefore \( \pi_1 \geq \pi_2 \geq \ldots \geq \pi_{n-1} \geq \pi_n \).

\(^{35}\) Indeed:

\[
\frac{d\pi_i}{ds_i} = \frac{dx_i}{ds_i} \left[ -x_ip'(X) \right] + x_is_i > 0
\]
marginal firm must expect zero profits, and that the associated zero profit condition determines the number of firms \( n \) that are active in the market: 
\[
x_n \left[ p(X) + s_n \right] - c(x_n) = F.
\]
Notice that this and the optimality condition for the marginal firm determine uniquely the total production in the market \( X(s_n) \) and the equilibrium production of the marginal firm \( x_n(s_n) \) as functions of the critical subsidy \( s_n \). Moreover, independently from the subsidies chosen by all the other countries, the equilibrium price must satisfy:
\[
p(X(s_n)) = c'(x_n) - x_n p'(X) - s_n = \frac{c(x_n) + F}{x_n} - s_n
\]  
which can be verified to be a decreasing function of \( s_n \).\(^{37}\) As a consequence, the equilibrium production of each firm \( i = 1, 2, \ldots, n - 1 \) must be a function of the subsidies of country \( i \) and \( n \) only, while the full set of subsidies determines the number of active firms.

Let us move to stage 1). To characterize the equilibrium we need to find a set of subsidies such that each one maximizes the welfare of the corresponding country taking as given the other subsidies and the equilibrium of the subgame. First of all, we can easily exclude the existence of a symmetric equilibrium where all countries with active firms adopt the same policy. Contrary to the Brander and Spencer (1985) case of an exogenous market structure, in our framework a common positive subsidy for all countries would lead to an endogenous market structure where all firms obtain zero profits: then, a country would prefer to deviate unilaterally and decrease its subsidization to reduce the costs needed to finance the subsidy. Also a common export tax (a negative subsidy) cannot be part of an equilibrium: otherwise a country would find it optimal to deviate and subsidize exports unilaterally. This implies that any equilibrium must entail some differentiation between the subsidies received by the active firms.

Given this, we can conjecture that in equilibrium there is a marginal firm receiving a critical subsidy \( s_n \) and obtaining zero profits in the market, with all the firms \( i < n \)

\(^{36}\)We are neglecting the integer constraint on the number of firms. As usual, this is a good approximation as long as the equilibrium number of firms is large enough.

\(^{37}\)This can be derived by total differentiation using the second order condition for profit maximization. Notice that in equilibrium, the production of the marginal firm \( x_n(s_n) \) is increasing in its subsidy if and only if the inverse demand is convex, and is independent from the subsidy with a linear demand (as in the example in the text).
active with subsidies larger than \( s_n \), and all the firms \( i \in (n, m] \) not active with subsidies smaller or equal to \( s_n \). We can now characterize the subgame perfect equilibrium selecting the welfare maximizing strategies of each country given the strategies of the others and the equilibrium of the subgames.

Consider first a country \( i < n \), whose firm is active in the market. Such a country maximizes its own welfare function:

\[
W(s_i, s_n) = x_i(s_i)p(X(s_n)) - c(x_i(s_i)) - F
\]  

(40)

where \( s_j \) for any \( j \neq i \) is taken as given. Notice that the equilibrium price depends only on the critical subsidy \( s_n \), and the production of the domestic firm depends on the domestic policy \( s_i \) and on the critical subsidy \( s_n \). The latter is taken as given. The optimal domestic policy must satisfy the first order condition \( p(X(s_n)) = c'(x_i(s_i)) \), which is the same for any country \( i < n \). This implies that each one of these countries adopts the same subsidy. Using the profit maximizing conditions, this can be rewritten as:

\[
s^*(s_n) = \frac{p(X(s_n))}{\epsilon} > 0
\]  

(41)

where \( \epsilon \) is the demand elasticity. Notice that in case \( s_n = 0 \) this boils down to the optimal unilateral export subsidy (15), while an increase in the critical subsidy \( s_n \) reduces the optimal subsidy \( s^* \).

Consider now a country \( i \geq n \) adopting a subsidy such that the national firm is either marginal in the market or not active at all. First, we claim that any equilibrium must have \( s_i \leq 0 \) for any \( i \geq n \). Suppose \( s_n > 0 \), with \( s^*(s_n) \geq s_n \geq s_{n+1} \) by construction. The firm of country \( n \) would be still the marginal firm obtaining zero net profits, but welfare would be negative because of the cost of the positive subsidy. Then, country \( n \) could be better off choosing free trade, that is \( s_n = 0 \).

Second, we claim that in equilibrium it must be exactly \( s_i = 0 \) for any \( i \geq n \). To verify this, first we show that this is an equilibrium, then we show that there cannot be another equilibrium.

Consider the candidate equilibrium with free trade for all countries \( i \geq n \). We prove that there are not profitable deviations for any country \( i \geq n \). First, we show that a unilateral deviation with a positive subsidy \( \hat{s} \geq s^*(0) \) cannot be profitable. This implies that the national firm will be active and endogenous entry will be binding on all the firms subsidized at the rate \( s^*(0) \). These firms will obtain zero net net
profits. Imposing such a zero profit condition, their equilibrium condition for profit maximization will satisfy:

\[ p(X(s^*(0))) = c'(x_j(s^*(0))) \text{ for } j = 1, 2, ..., n - 1 \]

Moreover, welfare for their countries will be negative because of the cost of the positive subsidy. Given this, the optimal deviation \( \hat{s} \geq s^*(0) \) must maximize:

\[ W(\hat{s}, s^*(0)) = x_i(\hat{s})p(X(s^*(0))) - c(x_i(\hat{s})) - F \]

where the price is taken as given. The optimal deviation follows the optimality condition \( p(X(s^*(0))) = c'(x_i(\hat{s})) \), which can only be satisfied by \( \hat{s} = s^*(0) \). Therefore the best deviation leads only to a reduction in welfare compared to free trade. Second, let us consider a deviation with the adoption of a subsidy \( \hat{s} \in (0, s^*(0)) \). If this induces the national firm to be active, it turns it into a marginal firm with zero profits, therefore national welfare is again negative because of the cost of the subsidy. As a consequence, there are no profitable deviations with positive subsidies for any country \( i \geq n \). Finally, we show that any country \( i \geq n \) cannot gain from a unilateral deviation with a negative subsidy. This would lead the national firm not to enter, therefore welfare would be the same as under free trade. By assumption, free trade is chosen when a larger welfare level cannot be reached.

To conclude the proof, we need to show that there cannot be another equilibrium. This would need at least a negative subsidy. Suppose that \( s_n < 0 \) in equilibrium. Then, a country \( j > n \) (without an active firm) could deviate unilaterally and choose \( \hat{s} = s_n + \varepsilon < 0 \) with \( \varepsilon \) positive and small enough, so as to increase welfare. The reason is that the national firm would then enter in the market, become the marginal firm with zero profits, and generate revenues thanks to the export tax. Suppose now that \( s_j < 0 \) for some \( j > n \). Then, the firm of country \( j \) would not enter in the market and welfare would be the same as under free trade for country \( j \). Therefore, this negative subsidy cannot belong to an equilibrium under our assumption that indifferent countries choose free trade.\(^{38}\) This concludes the proof.

\(^{38}\)Relaxing the assumption that indifferent countries choose free trade, the equilibrium implies at least \( s_n = s_{n+1} = 0 \). If \( s_n < 0 \) another country would find it profitable to deviate and adopt a smaller export tax (inducing the entry of its firm and gaining from the tax revenues). If \( s_{n+1} < 0 \) the marginal country would find it profitable to deviate and adopt an export tax slightly smaller than the one of country \( n + 1 \) (raising revenues from the export tax).
REFERENCES


