DEPARTMENT OF ECONOMICS

UNIVERSITY OF MILAN - BICOCCA

WORKING PAPER SERIES

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Marco Faravelli and Luca Stanca

No. 127 – December 2007

Dipartimento di Economia Politica
Università degli Studi di Milano - Bicocca
http://dipeco.economia.unimib.it

Single versus Multiple Prize Contests to Finance Public Goods: Theory and Experimental Evidence*

Marco Faravelli† and Luca Stanca‡

December 2007

Abstract

This paper investigates single and multiple prize contests as incentive mechanisms for the private provision of public goods, under the assumptions of income heterogeneity and incomplete information about income levels. We compare experimentally a one-prize contest with a three-prize contest in a case where theory predicts that several prizes maximise revenues. We find that, contrary to the theoretical predictions, total contributions are significantly higher in the one-prize contest. In both treatments contributions converge towards theoretical predictions over successive rounds, but the effects of repetition are different: convergence is fast in the one-prize treatment, while gradual and with some undershooting in the three-prize treatment. Focusing on individual income types, the better performance of the single-prize contest is largely explained by the contributions of high-income individuals: a single larger prize provides a more effective incentive for richer individuals than three smaller prizes.

Keywords: Auctions; Public Goods; Laboratory Experiments.
JEL codes: C91; D44; H41.

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*We thank Luca Corazzini for the experimental sessions. We acknowledge financial support from Econometica.

†School of Economics and Finance, University of St Andrews, UK. E-mail: marco.faravelli@st-andrews.ac.uk, http://www.st-andrews.ac.uk/∼mf60.

‡Economics Department, University of Milan Bicocca. Piazza dell'Ateneo Nuovo 1, 20126 Milan, Italy. E-mail: luca.stanca@unimib.it, http://dipeco.economia.unimib.it/Persone/Stanca.

1 Introduction

Prizes are commonly used as incentives in various spheres of human activity, from sports to education and research. The economic literature has analyzed extensively the use of contests as incentive mechanisms in different areas, such as rent-seeking activities, technological races and compensation schemes in labour markets, among others (see, as examples, Tullock, 1980; Lazear and Rosen, 1981; Broecker, 1990; Taylor, 1995; Fullerton and McAfee, 1999). The present study focuses on the application of contests as incentive devices for the private provision of public goods.

A number of recent papers have investigated, both theoretically and through the use of experiments, the use of prize-based mechanisms (either stochastic or deterministic) to incentivize contributions to public goods, showing that they can be an effective way to overcome free riding (e.g. Morgan, 2000; Morgan and Sefton, 2000; Goeree et al., 2005; Orzen, 2005; Schram and Onderstal, 2007; Faravelli, 2007; Corazzini et al., 2007). A fundamental question, related to the use of contests, is the optimal allocation of prizes. For a given total prize sum, is the award of a single prize more effective than multiple prizes? Or is the opposite true? As Moldovanu and Sela (2001, p. 543) put it: "The award of a single prize seems consistent with a general intuition about the efficiency of rewarding only the best (and supposedly ablest) competitor. But, the prevalence of multiple-prize contests is obvious in the real world".

Several theoretical studies have analyzed multiple-prize contests. The relative efficiency of single and multiple-prize contests depends on the specific setting of the model. Expected effort is generally independent of the number of prizes, for a given total prize, in symmetric settings. Barut and Kovenock (1998) analyze symmetric multiple-prize all-pay auctions with complete information. They focus on risk-neutral, unconstrained agents and show that only mixed strategy equilibria exist. Expected expenditures are maximized by driving the value of the lowest prize to zero, but are invariant across all configurations of prizes that leave the lowest prize equal to zero. Faravelli (2007) studies multiple-prize contests as a means to finance public goods, with risk neutral agents and symmetric linear cost, but asymmetric endowments. Incomes are drawn from a continuous distribution function and are private information. Asymmetry and incomplete information enable to characterize a monotone equilibrium, in which the contribution is strictly increasing in the endowment. As in Barut and Kovenock (1998), it is optimal to set the last prize equal to zero, but total expected contribution is independent of the distribution of the total prize sum among the prizes.

Multiple prizes may maximize total exerted effort when either risk-aversion (e.g. Glazer and Hassin, 1988) or some form of asymmetry is introduced. Moldovanu and Sela (2001) study a multiple-prize contest where unconstrained agents differ in the ability to exert effort, and the ability is private information. When costs are either linear or concave, a high ability individual will invest more in the contest allocating only one prize, that maximizes the total expected effort exerted by the bidders. However, when costs are convex this is not necessarily the case and multiple prizes could be optimal. Szymanski and Valletti (2005) consider a contest where three unconstrained players, who differ in their ability, compete for prizes. Ability is common knowledge and marginal costs are constant. In order to maximize total expected effort the third prize must be set equal to zero. However, it is not obvious how to optimally share the total prize sum between the first and second prize. The authors focus on two scenarios. In the case of two (equally) strong players and one weak agent, one prize maximizes total effort. This is because the weak player is bound to lose and let the strong agents compete between themselves, bidding zero. However, when one strong player faces two weak opponents (with equal ability), two prizes are optimal, because the second prize increases the competition between the weak agents.

Limited empirical evidence is available on the relative performance of single- and multiple-prize contests. Schmidt et al. (2005) compare the performance of three different stochastic contests through the use of a laboratory experiment: a single prize, a three-prize and a proportionate-prize lottery, with equal total prize.[1] If the players are risk neutral, the three treatments are equivalent. However the proportionate-prize lottery raises higher revenue than the multiple prize, which outperforms the single prize treatment, indicating that the subjects may be risk averse. Landry et al. (2006) conduct a field experiment on charity giving, comparing voluntary contributions, single and multiple prize lotteries (where an agent may win more than one prize). Lotteries increase participation rates by 100 percent compared to voluntary contributions. Further, multiple-prize lotteries raise slightly less revenue than single-prize.

Overall, the optimal allocation of prizes in contests is virtually unexplored from an experimental perspective, and the existing results are not conclusive. Furthermore, while distinct theoretical models provide different reasons to explain why more prizes may be optimal, from an empirical perspective it is not clear which characteristics may drive this result.

This paper presents an experimental analysis of single and multiple-prize contests as incentive mechanisms for the private provision of public goods. We compare two treatments in a between-subject design, focusing on two extreme scenarios, for a given total prize: a one-prize contest (winner takes all) and a three-prize contest (three highest bid receive an equal prize).[2] We focus on a setting characterized by asymmetric agents, in which theory predicts that multiple

[1] A subject's bid entitles him to lottery tickets. In the first treatment one ticket is randomly drawn and the holder wins the prize; in the second treatment three tickets are randomly drawn and the holders win the prizes (notice that a subject may win more than one prize); in the last treatment each subject is awarded a share of the prize equal to the proportion of his tickets relative to those of the entire group.

[2] In our design, contrary to Schmidt et al. (2005) and Landry et al. (2006), a subject cannot win more than one prize.

prizes should be more effective than a single prize. This setting follows Faravelli (2007), assuming income heterogeneity and incomplete information about income levels[3]. However, we introduce a discrete, rather than a continuous, distribution of incomes. This assumption has a similar effect as the introduction of asymmetry in Szymanski and Valletti (2005), and it implies that total expected contributions are maximized by awarding more than one prize.

At the theoretical level, we show that, with a single prize, an asymmetric equilibrium arises in which contributions are increasing with income. In addition, the asymmetry of incomes may be crucial for the optimal allocation of prizes: while it is still optimal to set the last prize equal to zero, if the prize is large enough the single-prize contest presents a loss of revenue due to the discontinuity in the possible endowments. On the other hand, in the multiple-prize mechanism the symmetric equilibrium identified by Barut and Kovenock (1998) emerges. All agents behave symmetrically and several prizes raise more revenue than a single prize.

Empirically, our main finding is that, contrary to the theoretical predictions, total contributions are significantly higher in the one-prize contest. Focusing on individual income levels, the better performance of the single-prize contest is largely explained by the contributions of high-income individuals. In both mechanisms, contributions tend to converge towards the theoretical predictions over successive rounds, but the effects of repetition are different: convergence is fast with one prize, while gradual and with some undershooting with three prizes.

In order to interpret the results on the relative efficiency of the two mechanisms we also compare empirical and theoretical contributions within each treatment. We find that in the single-prize contest contributions are significantly higher than theoretical predictions both on aggregate and, disaggregating by income types, at the lower end of the income distribution. In the three-prize contest contributions are not significantly different from the theoretical predictions over the 20 periods, while in the final 5 rounds they are lower than predictions on aggregate and at both ends of the income distribution.

The rest of the paper is organized as follows. Section 2 describes the experimental design and procedures. Section 3 presents the theory and the predictions for our design. Section 4 describes the experimental results. Section 5 concludes with a summary of the main findings and a discussion of the implications of the analysis.

2 Experimental Design

The experiment is based on a standard linear public good game and compares two treatments in a between-subject design: in the first treatment contributions to the

[3]Contrary to Moldovanu and Sela (2001) and Szymanski and Valletti (2005), in our model asymmetry is not in the ability but in the budget constraint.

public good are incentivized with a one-prize contest (1PC), while in the second treatment the incentive mechanism is a three-prize contest (3PC). The total prize sum is the same across the two treatments. The design is similar to the one used in Orzen (2005), while introducing income heterogeneity and incomplete information about the income of other subjects.

We ran three sessions for each treatment, with sixteen subjects participating in each session, for a total of 96 subjects. Each session consisted of 20 rounds. In each round, subjects played a linear public good game, choosing how to allocate their endowment between an individual and a group account.[4] In both treatments, a subject received 2 points for each token allocated to the individual account, while he received 1 point for each token allocated by him or by any other member of his group to the group account. The incentive mechanisms in the two treatments implied the same financial commitment (total prize) for the fundraiser, but differed in the way prizes (extra points) could be earned by subjects. In 1PC, in each round the member of the group who allocated the highest amount to the group account won the single prize of 240 points. In 3PC, in each round the three group members who allocated the highest amounts to the group account won a prize of 80 points each. In both treatments, in case of ties among one or more group members, the winner was determined randomly.

At the beginning of each session the sixteen subjects were randomly and anonymously assigned an endowment of either 120, 160, 200, or 240 tokens. Subjects were informed that in each round each subject would receive the same endowment as determined at the beginning of the session. Incomplete information about other subjects' endowments was introduced by adopting a strangers matching rule, as in Andreoni (1998). At the beginning of each round, subjects were randomly and anonymously rematched in groups of four. Therefore, in each round subjects did not know the identity and the endowment of the other three members of their group. They only knew that the endowment of each of the other group members could be either 120, 160, 200, or 240 tokens with equal probabilities.

Group matching for each of the twenty rounds was determined randomly before the beginning of the experiment in the following way. Four pools of four subjects were formed, each containing the four different income types (120, 160, 200, 240). Each of the four groups was formed by randomly drawing one subject from each pool. As a consequence, within every group each member could have an endowment of 120, 160, 200, or 240 tokens with equal probability.[5] Having formed the four groups for each round in this way, the same sequence of group matchings for the twenty rounds was used in each session of both treatments.

The experiment was run in May 2006 at the Experimental Economics Lab of

[4]The language used in the instructions did not refer to contributions or public goods, but asked subjects to allocate tokens to either an "individual account" or a "group account".

[5]Note that in every round there were four subjects for each of the four possible endowments, so that the average endowment was 180 tokens.

the University of Milan Bicocca.[6] In each session, subjects were randomly assigned to a computer terminal at their arrival. To ensure public knowledge, instructions were distributed and read aloud (see Appendix A for the instructions). Moreover, to ensure individual understanding of the public good game and the incentive mechanism, sample questions were distributed and the answers privately checked and, if necessary, explained to the subjects.

At the end of each round, subjects were informed about their payoffs from the group account, the individual account and from the prize. At the end of the last round subjects were informed about their total payoff expressed in points and euros. They were asked to answer a short questionnaire on the individual understanding of the experiment and socio-demographic information, and were then paid in private using an exchange rate of 1000 points per euro. On average subjects earned 12.25 euros for sessions lasting about 50 minutes, including the time for instructions. Participants were mainly undergraduate students of Economics and were recruited through an on-line system.

3 Theory and Predictions

This section presents the theoretical predictions for the two experimental treatments. Concerning the single prize contest, we study a linear public good game financed through an all-pay auction in which one prize is awarded, assuming that players have heterogeneous endowments and information is *complete*. We solve the game for N players, who can have any possible endowment, for any positive level of prize. We show that when the prize is not too high, only mixed strategy equilibria exist. An asymmetric equilibrium arises in which contributions are increasing with income. The equilibrium for a subject under *incomplete* information about the incomes of other players consists of a randomization over the mixed strategies he would play in all the possible group configurations he faces, according to their corresponding probabilities. With regard to the three-prize contest we solve the game of incomplete information played by the subjects in our experiment. We show that, if the prize is not too high, there exists a unique symmetric mixed strategy equilibrium in which all agents choose their contribution from the same distribution function.

3.1 Single-Prize Contest

Consider N players and the set of endowments $Z = (z_1, ..., z_S)$ such that $0 < z_1 < \ldots < z_S$. Each player has an endowment which takes value from the set Z. Call $n[z_i]$ the number of players with endowment $z_i \in Z$ such that $\sum_{i=1}^{S} n[z_i] = N$. The players' endowments and their number are common knowledge. With no loss of

[6] The experiment was computerized using the z-Tree software (Fischbacher, 2007).

generality, assume that $n[z_i] \geq 0$ for $1 \leq i \leq S-1$ and $n[z_S] \geq 1$. Players play a public good game in which each individual has to choose how much to contribute to the public good. At the same time they take part in an all-pay auction in which a prize $\Pi > 0$ is awarded to the agent who contributes the most. The bidders are risk-neutral and they all value the prize equally.

The payoff for a player with endowment z_i who contributes g_i is given by

$$\beta(z_i - g_i) + \beta E[\Pi, g_i, g_{-i}] + g_i + G_{-i}$$

where G_{-i} represents the sum of all other players' contributions and $1 < \beta < N$.[7]

We divide our analysis in two parts: $n[z_S] > 1$ and $n[z_S] = 1$.

3.1.1 More than One Player with the Highest Endowment

We study first the case in which $n[z_S] > 1$. There exist three possible scenarios: the prize level can be "high", "medium" or "low". In the next two propositions, we show that if and only if the prize level is "high" there exists a quasi-symmetric pure strategy equilibrium, in which agents with the same endowment behave identically.[8]

Proposition 1 *When $n[z_S] > 1$ and $z_S \leq \frac{\beta\Pi}{n[z_S](\beta-1)}$, there exists a quasi-symmetric pure strategy equilibrium in which players with endowment z_S contribute their full endowment, while if there are other agents with lower endowments they all contribute 0.*

Proposition 2 *When $n[z_S] > 1$ and $\frac{\beta\Pi}{n[z_S](\beta-1)} < z_S$, there exist no quasi-symmetric pure strategy equilibria.*

If the prize level is "medium" only the agents with the highest endowment will submit non-zero bids.

Proposition 3 *When $\frac{\beta\Pi}{n[z_S](\beta-1)} < z_S < \frac{\beta\Pi}{\beta-1}$ and $n[z_S] > 1$ there exists a mixed strategy equilibrium in which:*

- *players with endowment z_S contribute their full endowment with probability p and with probability $1-p$ they choose their contribution from the distribution function $F(g) = (\frac{(\beta-1)g}{\beta\Pi})^{\frac{1}{n[z_S]-1}}$ on the interval $[0, a]$, such that $F(a) = 1-p$, where $a < z_S$ and p is the unique solution to the following equation $\frac{1-(1-p)^{n[z_S]}}{n[z_S]p} = \frac{(\beta-1)z_S}{\beta\Pi}$;*
- *players with endowments lower than z_S contribute 0.*

[7]Note that in our experiment $\beta = 2$.

[8]See Appendix B for the proofs of all the propositions contained in this section.

Finally, if the prize level is "low" only the players with endowments higher than $\frac{\beta\Pi}{\beta-1}$ will contribute positive amounts.

Proposition 4 *When $z_S \geq \frac{\beta\Pi}{\beta-1}$ and $n[z_S] > 1$, there exists a mixed strategy equilibrium in which:*

- *players with endowment $z_i \geq \frac{\beta\Pi}{\beta-1}$ choose their contributions from the distribution function $F(g) = (\frac{(\beta-1)g}{\beta\Pi})^{\frac{1}{m-1}}$ on the interval $[0, \frac{\beta\Pi}{\beta-1}]$, where m is the number of players with endowment greater or equal than $\frac{\beta\Pi}{\beta-1}$;*
- *all other players contribute 0.*

3.1.2 Only One Player with the Highest Endowment

We look now at the case where $n[z_S] = 1$. First we will prove that only mixed strategy equilibria exist.

Proposition 5 *When $n[z_S] = 1$ there exist no pure strategy equilibria.*

There exist two possible cases: when the prize level is "low" and when it is "high". Let us start focusing on the first scenario.

Proposition 6 *When $z_{S-1} \geq \frac{\beta\Pi}{\beta-1}$ and $n[z_S] = 1$, there exists a mixed strategy equilibrium in which:*

- *players with endowment $z_i \geq \frac{\beta\Pi}{\beta-1}$ choose their contributions from the distribution function $F(g) = (\frac{(\beta-1)g}{\beta\Pi})^{\frac{1}{m-1}}$ on the interval $[0, \frac{\beta\Pi}{\beta-1}]$, where m is the number of players with endowment greater or equal than $\frac{\beta\Pi}{\beta-1}$;*
- *all other players contribute 0.*

When the prize level is "high", specifically $z_{S-1} < \frac{\beta\Pi}{\beta-1}$, if the strategy space is continuous, and ties are broken by randomly assigning the prize to one player, then no equilibrium exists. In order to avoid this problem, given that we are interested in the theoretical predictions of an experiment, where the strategy space is discrete, we will assume that there exists a smallest currency unit strictly above z_{S-1} (see Che and Gale, 1997).[9]

Proposition 7 *When $z_{S-1} < \frac{\beta\Pi}{\beta-1}$ and $n[z_S] = 1$, there exists a mixed strategy equilibrium in which:*

[9]The non-existence of the equilibrium is due to a discontinuity in the payoffs. Another way to avoid this problem would be to always break ties in favour of the player with the higher budget.

- *the player with endowment z_S chooses his contribution from the distribution function $H(g) = \frac{(\beta-1)g}{(\beta\Pi)^{\frac{1}{n[z_{S-1}]}}(\beta\Pi-(\beta-1)(z_{S-1}-g))^{\frac{n[z_{S-1}]-1}{n[z_{S-1}]}}}$ on the interval $[0, z_{S-1}]$ and puts a mass equal to $\frac{\beta\Pi-(\beta-1)z_{S-1}}{\beta\Pi}$ on the smallest currency unit strictly above z_{S-1};*

- *players with endowment z_{S-1} contribute zero with probability $(\frac{\beta\Pi-(\beta-1)z_{S-1}}{\beta\Pi})^{\frac{1}{n[z_{S-1}]}}$ and choose their contribution from the distribution function*

  $L(g) = (\frac{\beta\Pi-(\beta-1)(z_{S-1}-g)}{\beta\Pi})^{\frac{1}{n[z_{S-1}]}}$ *on the interval $(0, z_{S-1}]$;*

- *all other players contribute zero.*

3.2 Three-Prize Contest

Consider the case where contributions are incentivized through an all-pay auction where three equal prizes are awarded to the three agents who contribute the most. As in the previous case, the players' endowments are private information and are drawn from a discrete distribution which is common knowledge. The payoff for a player with endowment z_i who contributes g_i is given by

$$\beta(z_i - g_i) + \beta E[\pi, g_i, g_{-i}] + g_i + G_{-i}$$

where π is the value of each of the three prizes. We derive the equilibrium of the game in which four players take part and the lowest possible endowment is greater than $\frac{\beta\pi}{\beta-1}$. Let us first prove the following proposition.

Proposition 8 *There exist no symmetric pure strategy equilibria.*

Proof. Notice first that any contribution $g > \frac{\beta\pi}{\beta-1}$ is dominated by $g = 0$. Suppose that there exists a symmetric pure strategy equilibrium in which every agent plays $g \in [0, \frac{\beta\pi}{\beta-1})$. Then agent i would have an incentive to increase his bid by ε arbitrarily close to zero and win one prize for sure. Suppose now that there exists a symmetric pure strategy equilibrium where all players contribute $g = \frac{\beta\pi}{\beta-1}$. Then player i would have an incentive to bid zero. ■

Given that no symmetric equilibria exist in pure strategies, we turn our attention to mixed strategy equilibria.

Proposition 9 *There exists a unique symmetric mixed strategy equilibrium in which all agents choose their contribution from the distribution function $F(g) = \sqrt[3]{\frac{g}{\pi} - \frac{g}{\beta\pi} - 1} + 1$ on the interval $[0, \frac{\beta\pi}{\beta-1}]$.*

Proof. Any contribution $g > \frac{\beta\pi}{\beta-1}$ is dominated by $g = 0$. Because the lowest possible endowment is greater than $\frac{\beta\pi}{\beta-1}$, although each player's endowment is private information, the game is equivalent to a game with complete information in which none of the players is constrained.

Assume that three agents choose their contributions from the distribution function $F(g)$ on the interval $[0, \frac{\beta\pi}{\beta-1}]$. In order for this to be an equilibrium the remaining player i must be indifferent to play any $g \in [0, \frac{\beta\pi}{\beta-1}]$. Hence his expected payoff from playing g must be

$$\begin{aligned} & \beta z_i + \beta\pi((F(g))^3 + 3(F(g))^2(1 - F(g)) + 3F(g)(1 - F(g))^2) \\ -(\beta - 1)g + G_{-i} \quad = \quad & \beta z_i + G_{-i} \end{aligned}$$

The above expression can be rewritten as

$$\beta\pi F(g)\left((F(g))^2 - 3(F(g)) + 3\right) = (\beta - 1)g$$

This equation has a unique real root given by $F(g) = \sqrt[3]{\frac{g}{\pi} - \frac{g}{\beta\pi} - 1} + 1$. ■

3.3 Hypotheses

The theoretical predictions for the two treatments of the experiment, based on the results in the previous sections, are reported in Table 1. The single-prize contest has a quasi-symmetric mixed strategy equilibrium, in which agents with the same income behave identically. Expected contributions are increasing in the subjects' incomes and the average total contribution is 203 tokens. The three-prize contest has a unique symmetric mixed strategy equilibrium. Recall that in the 3PC treatment $\pi = 40$. Hence, in equilibrium all agents choose their contribution from the distribution function $F(g) = \sqrt[3]{\frac{g}{80} - 1} + 1$ on the interval $[0, 80]$. The expected individual contribution is 60 for all subjects, and the average total contribution is 240 tokens.

The different features of the equilibria in the two treatments can be explained as follows. In the single-prize contest the prize is sufficiently high for any player to randomize over his full support. This implies that subjects are differently constrained and in equilibrium agents with higher incomes contribute more than those who have lower incomes. In the three-prize contest bidding zero dominates any contribution greater than 80 tokens. Because any subject can afford to contribute up to 80, the game is actually a symmetric one and has a completely symmetric equilibrium.

The asymmetry in the equilibrium for the single-prize contest also explains why total expected contribution is lower in 1PC than in 3PC. Under a discrete income distribution, in the single-prize contest a subject with an endowment higher than the lowest one will face opponents with a strictly lower endowment with positive probability. In this case he will not have any incentive to bid more than the highest of his opponents' endowments, resulting in a loss of revenue with respect to the game with a continuous distribution.[10] This is obviously not the case for the three-prize contest, where none of the agents is constrained.

Summing up, the experiment is designed to test the following hypotheses:

H1 Total contributions to the public good are higher with three prizes than with one prize.

H2 Average contributions to the public good are higher with three prizes than with one prize for low-income individuals, while the opposite holds for high-income individuals.

H3 Average contributions are steeply positively related to income in the one-prize contest, while independent of income in the three-prize contest.

4 Results

This section presents the experimental results. We start with a comparison of the two treatments in terms of average contributions and disaggregating by income level. Next, in order to interpret the results on the relative efficiency of the two mechanisms, we compare empirical and predicted contributions within each treatment, considering average contributions both over all subjects and by individual income types. Finally, we examine the experimental data at individual-level.

4.1 Comparison between treatments

Figure 1 displays average contributions over rounds for the two treatments. In both treatments average contributions tend to converge to the theoretical predictions over successive rounds, but the effects of repetition are quite different. In the 1PC contributions decline in the first 6 rounds, then remain relatively stable well above the average predicted level (50.8). In the 3PC contributions fall steadily throughout the first 18 rounds, to then stabilize well below the average predicted level (60).

Table 2 reports individual contributions, by treatment, averaging over all subjects and by individual endowment types, over all 20 rounds and breaking down the sample into individual sub-periods (rounds 1-5, 6-10, 11-15, and 16-20). Overall, contrary to the theoretical predictions, the 1PC does better than the 3PC: considering all rounds and income types, average contributions are 72 tokens in 1PC and 64.2 tokens in 3PC. This result is indeed much stronger if we consider the final rounds, after there has been some learning. In the last five rounds average contributions are 65.3 tokens in the 1PC and 41.4 tokens in the 3PC .

[10] With a continuous income distribution the two contests would raise the same total expected revenue of 240 tokens (see Faravelli, 2007).

The difference in contributions between the two incentive mechanisms is statistically significant. Table 3 presents Wilcoxon rank-sum tests of the null hypothesis that contributions to the public good are the same across treatments.[11] Given that the theoretical model predicts the direction of departure from the null hypothesis (e.g. 3PC>1PC averaging over all income types), we use the relevant one sided-tests. Considering all income types and the whole sample the test statistic is negative but not significant, owing to the different effects of repetition in the two treatments. However, focusing on the last ten or the last five rounds, test statistics are negative and strongly significant. Overall, the results indicate that, contrary to the theoretical predictions, average contributions to the public good are higher in 1PC than in 3PC.

> **Result 1**: The one-prize contest generates significantly higher contributions to the public good than the three-prize contest.

So far we have considered average contributions over all subjects. We now focus on individual income levels and compare the two mechanisms at different ends of the income distribution. Figure 2 compares the two incentive mechanisms for each income level. In both treatments contributions depend positively on incomes. However, while average contributions are steeply increasing in income in the 1PC, they are weakly related to income in 3PC. As a result, average contributions are relatively similar across treatments for incomes up to 200, whereas they are very different for high-income individuals (240). This indicates that a single (larger) prize provides a more effective incentive for richer individuals than three smaller prizes. The lower average contribution in the 3PC is largely explained by the low contributions of the highest income type. Figure 3 compares the two mechanisms by endowment level over successive sub-periods. The graphs indicate that while the two mechanisms display similar profiles in the initial rounds, over successive rounds the single-prize contest tends to dominate the multi-prize contest throughout the income distribution. Indeed, the results in table 2 indicate that the 1PC significantly dominates 3PC for high-income types over the 20 rounds and for all income types in the final 5 rounds.

> **Result 2**: The one-prize contest provides a significantly more effective incentive mechanism than the three-prize contest for high-income individuals over the 20 rounds, and for all income levels in the last 5 rounds.

[11]Note that, because of the random rematching mechanism, independence of subject-level observations could be violated. However, the characteristics of the experimental design are such that the dependence across individual observations can be considered negligible, both theoretically and empirically (see Corazzini et al., 2007).

4.2 Actual and predicted contributions

Figure 4 compares empirical and predicted contributions within each treatment. As predicted by the theory, contributions are positively related to income in the 1PC, although not as steeply as in the theoretical prediction, while they are relatively flat in the 3PC. Subjects tend to over-contribute in 1PC for all income types, except the highest. In 3PC the high-income types over-contribute, while the contributions of subjects with the lowest income (120) are below the theoretical prediction. Tables 4 and 5 report results of sign tests of the null hypothesis that the empirical and theoretical contributions are the same within each treatment, over the twenty rounds or by sub-periods. Averaging over all twenty rounds, over-contributions in 1PC are statistically significant when aggregating over all-income types, and for low-income individuals (120 and 160). A similar pattern applies to individual sub-samples. The results for 3PC indicate that actual and predicted contributions are not significantly different, both on aggregate and by individual income type.

Figures 5 and 6 display empirical and predicted contributions by income type in each of the two treatments, separately for each sub-period. Focusing on the final rounds, the corresponding results from tables 3 and 4 indicate that contributions in 1PC are significantly higher than theoretical predictions for incomes 120 and 160. In 3PC contributions are significantly lower than theoretical predictions for incomes 120 and 240.

> **Result 3**: In the one-prize contest contributions are significantly higher than theoretical predictions at the lower end of the income distribution and on aggregate, both averaging over the 20 periods and by individual sub-periods. In the one-prize contest contributions are not significantly different from predictions over the 20 periods, while in the final 5 rounds they are lower than predictions on aggregate and at both ends of the income distribution.

4.3 Individual Observations

Finally, we compare the performance of the two mechanisms focusing on individual-level observations. Figures 7 and 8 present the histograms and the corresponding cumulative distribution functions of individual relative contributions for the two treatments.[12] The main difference between the two treatments is that in 1PC the distribution of contributions is more disperse, and subjects choose extreme values (0 and, to a lesser extent, 100%) much more often than in 3PC. However, contributions are clustered around low levels in 3PC. The cumulative distribution for 1PC lies above that for 3PC only up to a relative contribution of about 23 per

[12]Relative contributions are calculated as absolute contributions divided by the subject's income.

cent. Figures 9 and 10 present the same information on the distribution of individual observations separately for each income level. The same pattern observed at the aggregate level applies to individual income types. The differences between the distributions for the two treatments become more and more pronounced as we consider higher income levels.

5 Conclusions

This paper presented an experimental analysis of single and multiple-prize contests as incentive mechanisms for the private provision of public goods, under the assumptions of income heterogeneity and incomplete information about income levels. The main objective was to assess the relative performance of a one-prize contest and a three-prize contest, both at the aggregate level and by individual income type, in a setting where theory predicts that several prizes maximize contributions.

The key finding from the experimental analysis is that, contrary to theoretical predictions, total contributions are significantly higher in the one-prize contest. This result is even stronger when considering the final rounds of the experiment. In both mechanisms, contributions tend to converge towards the theoretical predictions over successive rounds, but the effects of repetition are different: convergence is fast with one prize while gradual and with some undershooting with three prizes. Focusing on individual income levels, the better performance of the single-prize contest is largely explained by the higher contributions of high-income individuals.

In order to interpret the results on the relative efficiency of the two mechanisms, we also compared empirical and predicted contributions within each treatment. We found that in the 1PC contributions are significantly higher than theoretical predictions at the lower end of the income distribution and on aggregate. In 3PC contributions are not significantly different from predictions over the 20 periods, while in the final 5 rounds they are lower than predictions on aggregate and at both ends of the income distribution. Finally, we compared the performance of the two incentive mechanisms focusing on individual-level observations. Overall, the distribution of individual contributions in the 1PC is characterised by a much higher fraction of extreme values, and in particular zero contributions, than in the 3PC. However, contributions tend to be clustered around low levels in the 3PC.

What can explain the main result that a single-prize mechanism is relatively more efficient than a multiple-prize mechanism? One possible interpretation is that a multiple-prize contest is a relatively less familiar mechanism, or can be perceived as more difficult, so that it provides a less effective incentive. Another explanation of the divergence between the results and the theoretical predictions could lie in the assumption of risk neutrality. To the extent that, with small amounts at stake, subjects tend to be risk lovers, the more egalitarian structure of the three-prize contest could provide a less effective incentive. A third possibility lies in the flat structure of prizes in the three-prize contest. We studied two extreme scenarios: winner-take-all versus a case where all bids except the lowest are awarded an equal prize. It is possible that removing the constraint of prizes being equal, a multiple-prize contest with differentiated prizes would provide a better incentive mechanism.

The results of our experiment indicate that a multiple-prize contest is less effective than a single-prize contest, for a given total prize, as an incentive mechanism for the private provision of public goods. This result is in contrast to the theoretical predictions of expected revenues being independent of the number of prizes in symmetric settings, or multiple-prizes raising higher revenues in asymmetric settings, as in our experimental design. Exploring the possible explanations of the rejection of the theoretical predictions will be the object of future research.

Appendix A: Instructions

Welcome. Thanks for participating in this experiment. If you follow the instructions carefully and make good decisions you can earn an amount of money that will be paid to you in cash at the end of the experiment. During the experiment you are not allowed to talk or communicate in any way with other participants. If you have any questions raise your hand and one of the assistants will come to you to answer it. The rules that you are reading are the same for all participants.

General rules

There are 16 people participating in this experiment. At the beginning of the experiment each participant will be assigned randomly and anonymously an endowment of either 120, 160, 200, or 240 tokens with equal probabilities.

The experiment will consist of 20 rounds. In each round you will have the same endowment that has been assigned to you at the beginning of the experiment. In each round you will be assigned randomly and anonymously to a group of four people. Therefore, of the other three people in your group you will not know the identity and the endowment, that could be 120, 160, 200, or 240 tokens with equal probabilities.

How your earnings are determined

In each round you have to decide how to allocate your endowment between an INDIVIDUAL ACCOUNT and a GROUP ACCOUNT, considering the following information:

- for each token that you allocate to the INDIVIDUAL ACCOUNT you will receive 2 points.
- for each token allocated to the GROUP ACCOUNT (by you or by any other of the members of your group), every group member will receive 1 point.

[**1PC**]

In each round you can win a prize of **240 points** on the basis of the following rules. The member of your group who allocates the highest amount to the GROUP ACCOUNT is the winner of the prize. In case of ties among one or more group members, the winner is determined randomly.

At the end of each round the computer will display how many tokens you have allocated to the two accounts and how many points you have obtained from each of the two accounts, from the prize, and in total. At the end of the experiment the total number of points you have obtained in the 20 rounds will be converted in Euros at the rate 1000 points = 1 Euro. The resulting amount will be paid to you in cash.

[**3PC**]

In each round you can win one of 3 prizes of **80 points** each on the basis of the following rules. The members of your group who allocate the three highest amounts to the GROUP ACCOUNT are the winners of the three prizes. In case of ties among one or more group members, the winner is determined randomly.

At the end of each round the computer will display how many tokens you have allocated to the two accounts and how many points you have obtained from each of the two accounts, from the prize, and in total. At the end of the experiment the total number of points you have obtained in the 20 rounds will be converted in Euros at the rate 1000 points = 1 Euro. The resulting amount will be paid to you in cash.

Appendix B

Proof of Proposition (1). If all players with endowment z_S contribute their full endowment each of them has an expected payoff of

$$\beta z_s + \frac{\beta \Pi}{n[z_S]} - (\beta - 1) z_S + G_{-i}$$

which is greater or equal than the payoff he could get from any other choice $g \in [0, z_S)$.[13]

If there are other players with lower endowments it is equally obvious that contributing 0 is for them a dominant strategy. ■

Proof of Proposition (2). In order to prove this it is enough to show that there exist no equilibria in which players with endowment z_S play according to the same pure strategy. The proof is in two parts.

i) Consider first the case in which $\frac{\beta\Pi}{n[z_S](\beta-1)} < z_S \leq \frac{\beta\Pi}{\beta-1}$. Suppose that players with endowment z_S contribute $g \in [0, z_S)$, then player i has an incentive to raise his own bid by an amount ε and win the prize. Equally, if all of them contribute z_S, then player i has an incentive to contribute 0.

ii) Consider now the case in which $z_S > \frac{\beta\Pi}{\beta-1}$. Notice first that any contribution $g > \frac{\beta\Pi}{\beta-1}$ is dominated by $g = 0$. Suppose that players with endowment z_S contribute $g \in [0, \frac{\beta\Pi}{\beta-1})$. Player i has an incentive to raise his own bid by an amount ε and win the prize. On the other hand if all of them contribute $g = \frac{\beta\Pi}{\beta-1}$, then player i has an incentive to deviate and contribute nothing. ■

Proof of Proposition (3). The proof is in five parts.

Let us first focus on the players with endowment z_S and show that, when they are the only active bidders, the candidate equilibrium is indeed an equilibrium.

i) Assume that all but one of the $n[z_S]$ players with endowment z_S choose their contribution from the distribution function $F(g) = (\frac{(\beta-1)g}{\beta\Pi})^{\frac{1}{n[z_S]-1}}$ on the interval $[0, a]$, where $0 < a < z_S$. Then the expected payoff of the remaining player i from contributing $g \in [0, a]$ is given by

$$\begin{aligned} & \beta z_S + \beta\Pi(F(g))^{n[z_S]-1} - (\beta - 1)g + G_{-i} \\ = \quad & \beta z_S + G_{-i} \end{aligned}$$

which is independent of g.

Assume now that $n[z_S] - 1$ players contribute their full endowment with probability p. Player i's expected prize from contributing z_S is given by

$$\beta\Pi \sum_{j=0}^{n[z_S]-1} \frac{1}{j+1} \binom{n[z_S]-1}{j} p^j (1-p)^{n[z_S]-j-1} \tag{1}$$

[13] In all the proofs G_{-i} represents the sum of all other agents' contributions.

where $\binom{n[z_S]-1}{j} p^j (1-p)^{n[z_S]-j-1}$ represents i's probability of tying with j other players, while $\frac{\beta\Pi}{j+1}$ is his expected prize when he ties with j others. Applying binomial rules expression (1) can be rewritten as

$$\beta\Pi\frac{1-(1-p)^{n[z_S]}}{n[z_S]p}$$

and therefore player i's expected payoff from playing z_S is given by

$$\beta z_S + \beta\Pi\frac{1-(1-p)^{n[z_S]}}{n[z_S]p} - (\beta-1)z_S + G_{-i}$$

For this to be an equilibrium player i's expected payoff from contributing z_S must be equal to his expected payoff from choosing any $g \in [0,a]$, which means that

$$\begin{aligned} & \beta z_S + \beta\Pi\frac{1-(1-p)^{n[z_S]}}{n[z_S]p} - (\beta-1)z_S + G_{-i} \\ = \; & \beta z_S + G_{-i} \end{aligned}$$

Therefore p must satisfy the following

$$\frac{1-(1-p)^{n[z_S]}}{n[z_S]p} = \frac{(\beta-1)z_S}{\beta\Pi} \tag{2}$$

ii) We are going to prove that there is a unique solution to equation (2). This equation can be rewritten as

$$1-(1-p)^{n[z_S]} = \frac{n[z_S](\beta-1)z_S}{\beta\Pi}p$$

Notice that the left hand side is concave while the right hand side is linear. Further, given the restrictions on z_S, it is the case that $1 < \frac{n[z_S](\beta-1)z_S}{\beta\Pi} < n[z_S]$. When $p=0$ both sides of the equation are equal to zero. When $p=1$ the left hand side is equal to 1 while the left hand side is strictly greater than 1. Finally, notice that the slope of the left hand side when $p=0$ is $n[z_S]$, which is steeper than the right hand side. Therefore there must be a unique solution for $p \in (0,1]$.

iii) We want to show that a, such that $F(a) = 1-p$, is strictly less than z_S. We will prove it by contradiction. Assume the opposite, then it should be the case that $F(z_S) \leq 1-p$. Given equation (2), the latter can be rearranged as

$$1-(1-p)^{n[z_S]} \leq n[z_S]p(1-p)^{n[z_S]-1} \tag{3}$$

When $p=0$ both sides are equal to 0. The first derivative of the left hand side is equal to $n[z_S]\,(1-p)^{n[z_S]-1}$, while the first derivative of the right hand side is $n[z_S]\,(1-p)^{n[z_S]-1} - (n[z_S]-1)n[z_S]p(1-p)^{n[z_S]-2}$. Notice that the former is strictly greater than the latter for any p on the interval $(0,1]$. Therefore the left hand side of inequality (3) is strictly greater than the right hand side for any positive probability, which contradicts our assumption.

iv) What we have just shown means that the players will not choose any contribution from the interval (a, z_S). Let us check that this is the case. Assume that all other players play according to the candidate equilibrium while player i contributes $g \in (a, z_S)$. Then i wins the prize with probability $(1-p)^{n[z_S]-1} = \frac{(\beta-1)a}{\beta\Pi}$ and his expected payoff is

$$\begin{aligned} & \beta z_S + \beta\Pi((\frac{(\beta-1)a}{\beta\Pi})^{\frac{1}{n[z_S]-1}})^{n[z_S]-1} - (\beta-1)g + G_{-i} \\ = \; & \beta z_S + (\beta-1)a - (\beta-1)g + G_{-i} \end{aligned}$$

which is strictly less than $\beta z_S + G_{-i}$. Therefore contributing 0 dominates any choice $g \in (a, z_S)$.

v) Let us now show that, when players with endowment z_S play according to the equilibrium candidate, it is a dominant strategy for all the other players to contribute nothing. Suppose that $z_{S-1} > a$. Point iv) proves that contributing 0 dominates any $g \in (a, z_{S-1}]$. On the other hand, if a player i with endowment $z_i < z_S$ contributes $g_i \in (0, a]$ then his expected payoff is

$$\beta z_i + \beta\Pi(\frac{(\beta-1)g_i}{\beta\Pi})^{\frac{n[z_S]}{n[z_S]-1}} - (\beta-1)g_i + G_{-i}$$

Given that

$$(\frac{(\beta-1)g_i}{\beta\Pi})^{\frac{n[z_S]}{n[z_S]-1}} < \frac{(\beta-1)g_i}{\beta\Pi}$$

it must be the case that contributing 0 is a dominant strategy for all players with endowment lower than z_S.

The same is true when $z_{S-1} \leq a$. ■

Proof of Proposition (4). Suppose that $z_{l-1} < \frac{\beta\Pi}{\beta-1}$ while $z_l \geq \frac{\beta\Pi}{\beta-1}$, with $1 \leq l \leq S$, and call $m = \sum_{i=l}^{S} n[z_i]$ the number of players with endowment greater or equal than $\frac{\beta\Pi}{\beta-1}$. If $l = 1$ then consider z_{l-1} to be zero. The proof is in four parts.

i) Notice first that any strategy above $\frac{\beta\Pi}{\beta-1}$is dominated by contributing 0.

ii) Let us focus on the interval $(z_{l-1}, \frac{\beta\Pi}{\beta-1}]$ where only m players are active. Assume that all but one of the m players choose their contribution from the distribution function $F(g)$ on the interval $(z_{l-1}, \frac{\beta\Pi}{\beta-1}]$. In order for this to be an equilibrium the remaining player i must be indifferent to play any $g \in (z_{l-1}, \frac{\beta\Pi}{\beta-1}]$. Hence his expected payoff from playing g must be

$$\beta z_i + \beta\Pi(F(g))^{m-1} - (\beta-1)g + G_{-i} = \beta z_i + G_{-i} + c$$

where $c \geq 0$.

This means that on the interval $(z_{l-1}, \frac{\beta\Pi}{\beta-1}]$ any player with endowment greater than z_{l-1} randomizes according to the following distribution function

$$F(g) = (\frac{(\beta-1)g + c}{\beta\Pi})^{\frac{1}{m-1}}$$

Note that $F(\frac{\beta\Pi}{\beta-1}) \leq 1$ implies that c must be equal to 0 and therefore we have a unique solution

$$F(g) = (\frac{(\beta-1)g}{\beta\Pi})^{\frac{1}{m-1}} \tag{4}$$

iii) Suppose that $l = 1$. When the other $N-1$ players choose their contribution from $F(g)$ on the interval $[0, \frac{\beta\Pi}{\beta-1}]$, then player i's expected payoff is equal to

$$\beta z_i + G_{-i}$$

independently of his contribution on the same interval.

iv) If $l > 1$ then point v) of the proof of Proposition (3) shows that contributing 0 is a dominant strategy for all players with endowment less than $\frac{\beta\Pi}{\beta-1}$, while players with higher endowments will randomize according to $F(g)$ from the interval $[0, \frac{\beta\Pi}{\beta-1}]$. ■

Proof of Proposition (5). The proof is in two parts.

i) Consider the case $z_{S-1} < \frac{\beta\Pi}{\beta-1}$. Suppose that there exists a pure strategy equilibrium characterised by the strategy profile $[g_1, \ldots, g_i, \ldots, g_N]$, where g_i is the contribution chosen by the generic player i. Call g_h the highest contribution. If $g_h > z_{S-1}$ then the player with endowment z_S could marginally lower his bid and increase his payoff. If $g_h < z_{S-1}$ then there is at least one player who could deviate and contribute $g_h + \varepsilon$, winning the prize and making a positive profit. If $g_h = z_{S-1}$ and a player with endowment z_{S-1} is contributing g_h, then the player with the highest endowment has an incentive to deviate and contribute $z_{S-1} + \varepsilon$. If $g_h = z_{S-1}$ and the players with endowment z_{S-1} are contributing strictly less than g_h, then the player with endowment z_S could lower his bid increasing his payoff.

ii) Consider the case $z_{S-1} \geq \frac{\beta\Pi}{\beta-1}$. Notice that any strategy $g > \frac{\beta\Pi}{\beta-1}$ is dominated by $g = 0$. As we have done above, suppose that there exists a pure strategy equilibrium characterised by the strategy profile $[g_1, \ldots, g_i, \ldots, g_N]$ and call g_h the highest contribution. If $g_h < \frac{\beta\Pi}{\beta-1}$ then there is at least one player who has an incentive to deviate and contribute $g_h + \varepsilon$. If $g_h = \frac{\beta\Pi}{\beta-1}$and only one player is contributing g_h, then he could lower his bid. If $g_h = \frac{\beta\Pi}{\beta-1}$ and two or more players are bidding g_h, then each one of them would be better off by contributing zero. ■

Proof of Proposition (6). Proof as in Proposition (4) ■

Proof of Propositio (7). Assuming that the players with budgets z_{S-1} and z_S are the only ones who submit positive bids, we show that by playing according to the equilibrium candidate they make each others indifferent between any possible choice. We then go on to prove that if they play in such a way it is a dominant strategy for all other players to contribute zero. The proof is in three parts.

i) Let us start supposing that the players with endowments strictly lower than z_{S-1} contribute zero. Note first that the player of type z_S can guarantee himself a positive surplus by submitting a bid above z_{S-1}. We want to show that if players with endowment z_{S-1} choose their contribution from $L(g)$, and play zero with probability $(\frac{\beta\Pi-(\beta-1)z_{S-1}}{\beta\Pi})^{\frac{1}{n[z_{S-1}]}}$, then the agent with the highest endowment is indifferent between any choice on the interval $(0, z_{S-1}]$. His payoff from playing $g \in (0, z_{S-1}]$ will be

$$\begin{aligned}
& \beta z_S + \beta\Pi(L(g))^{n[z_{S-1}]} - (\beta-1)g + G_{-i} \\
= \; & \beta z_S + \beta\Pi(\frac{\beta\Pi - (\beta-1)(z_{S-1}-g)}{\beta\Pi}) - (\beta-1)g + G_{-i} \\
= \; & \beta z_S + \beta\Pi - (\beta-1)z_{S-1} + G_{-i}
\end{aligned}$$

which indeed does not depend on g.

ii) Suppose now that the player with endowment z_S randomizes according to $H(g)$ on the interval $[0, z_{S-1}]$ and puts a mass equal to $\frac{\beta\Pi-(\beta-1)z_{S-1}}{\beta\Pi}$ on the smallest currency unit strictly above z_{S-1}.[14] If all other agents of type z_{S-1} play according to $L(g)$, and contribute zero with probability $(\frac{\beta\Pi-(\beta-1)z_{S-1}}{\beta\Pi})^{\frac{1}{n[z_{S-1}]}}$, then the payoff of a player with z_{S-1} from a choice $g \in [0, z_{S-1}]$ is given by

$$\begin{aligned}
& \beta z_S + \beta\Pi(L(g))^{n[z_{S-1}]-1}H(g) - (\beta-1)g + G_{-i} \\
= \; & \beta z_S + \beta\Pi(\frac{\beta\Pi - (\beta-1)(z_{S-1}-g)}{\beta\Pi})^{\frac{n[z_{S-1}]-1}{n[z_{S-1}]}} \\
& \frac{(\beta-1)g}{(\beta\Pi)^{\frac{1}{n[z_{S-1}]}}(\beta\Pi - (\beta-1)(z_{S-1}-g))^{\frac{n[z_{S-1}]-1}{n[z_{S-1}]}}} \\
& -(\beta-1)g + G_{-i} \\
= \; & \beta z_S + G_{-i}
\end{aligned}$$

which again is independent of g. It should be clear now why it is necessary to assume that there exists a smallest unit strictly above z_{S-1}. If this was not the case the player with the highest endowment would have a mass point at z_{S-1}. But then, if ties are broken by randomly assigning the prize to one player, an agent of type z_{S-1} would have an incentive to deviate and bid all his endowment.

iii) Finally, we want to show that if the agents of type z_{S-1} and z_S play as we described then it is a dominant strategy for all other players to contribute zero. If a player i with endowment $z_i < z_{S-1}$ contributes $g \in (0, z_i]$ his payoff is

[14] Note that, acording to $H(g)$, player z_S's bid is strictly positive and therefore no ties are possibile at zero.

represented by

$$\begin{aligned}
& \beta z_S + \beta\Pi(L(g))^{n[z_{S-1}]}H(g) - (\beta-1)g + G_{-i} \qquad (5)\\
= \; & \beta z_S + \beta\Pi(\frac{\beta\Pi - (\beta-1)(z_{S-1}-g)}{\beta\Pi}) \\
& \frac{(\beta-1)g}{(\beta\Pi)^{\frac{1}{n[z_{S-1}]}}(\beta\Pi - (\beta-1)(z_{S-1}-g))^{\frac{n[z_{S-1}]-1}{n[z_{S-1}]}}} - (\beta-1)g + G_{-i} \\
= \; & \beta z_S + (\frac{\beta\Pi - (\beta-1)(z_{S-1}-g)}{\beta\Pi})^{\frac{1}{n[z_{S-1}]}}(\beta-1)g - (\beta-1)g + G_{-i} \\
= \; & \beta z_S + (\beta-1)g((\frac{\beta\Pi - (\beta-1)(z_{S-1}-g)}{\beta\Pi})^{\frac{1}{n[z_{S-1}]}} - 1) + G_{-i}
\end{aligned}$$

On the other hand, if he plays $g = 0$ he gets a payoff equal to $\beta z_S + G_{-i}$. Note that $(\frac{\beta\Pi-(\beta-1)(z_{S-1}-g)}{\beta\Pi})^{\frac{1}{n[z_{S-1}]}} < 1$ and we conclude that expression (5) is strictly lower than $\beta z_S + G_{-i}$. ■

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Table 1: Theoretical predictions for the experiment

	Incomes				Total
Treatments	120	160	200	240	
1PC	5	28	68	102	203
3TPC	60	60	60	60	240

Note: contributions are rounded to the nearest integer.

Table 2: Average absolute contributions: by endowment and rounds

	Rounds				
Endowment	1 - 20	1-5	6-10	11-15	16-20
1PC-120	42.8	43.8	41.8	45.4	40.1
1PC-160	65.1	67.0	68.4	52.6	72.5
1PC-200	81.6	113.8	71.9	74.0	66.5
1PC-240	98.6	134.5	98.5	79.3	82.2
Average	72.0	89.8	70.2	62.8	65.3
3PC-120	49.8	70.9	57.3	42.1	29.0
3PC-160	60.4	77.4	66.0	52.9	45.5
3PC-200	81.4	111.1	86.2	73.3	55.0
3PC-240	65.3	126.7	68.7	29.6	36.0
Average	64.2	96.5	69.5	49.5	41.4

Note: contributions are expressed as number of tokens.

Table 3: Tests of equality between treatments by endowment

	Rounds				
Endowment	1-20	1-5	6-10	11-15	16-20
120	1.57	4.19	3.14	-1.05	-1.57
	(0.06)	(0.00)	(0.00)	(0.15)	(0.06)
160	-1.05	1.05	-0.52	0.26	-3.14
	(0.15)	(0.15)	(0.30)	(0.40)	(0.00)
200	-0.52	0.00	1.57	0.00	-3.66
	(0.30)	(0.50)	(0.94)	(0.50)	(0.00)
240	-4.19	0.00	-2.09	-3.66	-4.19
	(0.00)	(0.50)	(0.02)	(0.00)	(0.00)
Overall	-1.06	2.90	0.86	-1.88	-5.81
	(0.15)	(0.00)	(0.20)	(0.03)	(0.00)

Note: the table reports Wilcoxon rank-sum tests (normalized z-statistics) for the hypothesis that the median of the difference between individual contributions to the public good in the two treatments (3PC-1PC) is zero. P-values (in brackets), based on the standard normal distribution, refer to one-sided tests as predicted by the theory.

Table 4: Tests for predicted contributions: 1PC

	Rounds				
Endowment	1-20	1-5	6-10	11-15	16-20
120	1.00	0.83	0.92	1.00	1.00
	(0.00)	(0.04)	(0.01)	(0.00)	(0.00)
160	1.00	0.92	1.00	0.67	1.00
	(0.00)	(0.01)	(0.00)	(0.39)	(0.00)
200	0.58	0.83	0.50	0.42	0.58
	(0.77)	(0.04)	(1.00)	(0.77)	(0.77)
240	0.33	0.67	0.50	0.25	0.25
	(0.39)	(0.39)	(1.00)	(0.15)	(0.15)
Overall	0.73	0.81	0.73	0.58	0.71
	(0.00)	(0.00)	(0.00)	(0.31)	(0.01)

Note: the table reports results of sign tests of the hypothesis that the median of the empirical distribution is equal to the theoretical prediction. The statistics reported are the number of positive differences (as a fraction of the total, excluding ties). P-values (in brackets) refer to two-sided tests based on the binomial distribution.

Table 5: Tests for predicted contributions: 3PC

	Incomes				
Endowment	1-20	1-5	6-10	11-15	16-20
120	0.25	0.55	0.42	0.25	0.00
	(0.15)	(1.00)	(0.77)	(0.15)	(0.00)
160	0.33	0.55	0.42	0.33	0.33
	(0.39)	(1.00)	(0.77)	(0.39)	(0.39)
200	0.67	1.00	0.64	0.55	0.42
	(0.39)	(0.00)	(0.55)	(1.00)	(0.77)
240	0.50	1.00	0.42	0.08	0.09
	(1.00)	(0.00)	(0.77)	(0.01)	(0.01)
Overall	0.44	0.78	0.47	0.30	0.21
	(0.47)	(0.00)	(0.77)	(0.01)	(0.00)

Note: the table reports results of sign tests of the hypothesis that the median of the differences between empirical and theoretical individual contributions is zero. The statistics reported are the number of positive differences (as a fraction of the total, excluding ties). P-values (in brackets) refer to two-sided tests based on the binomial distribution.

Figure 1: Average contributions over time

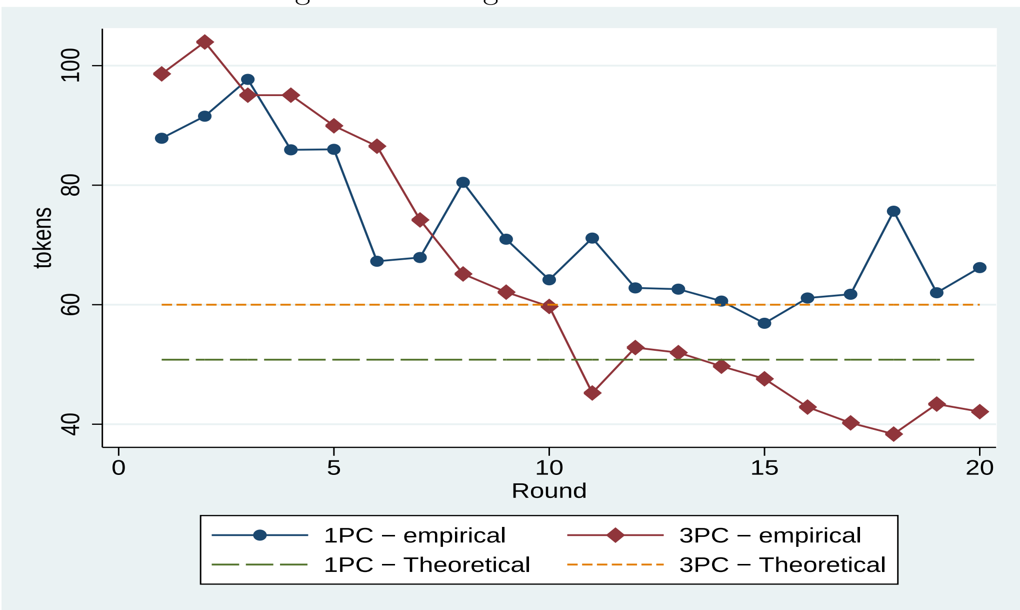

Figure 2: Average contributions by endowment

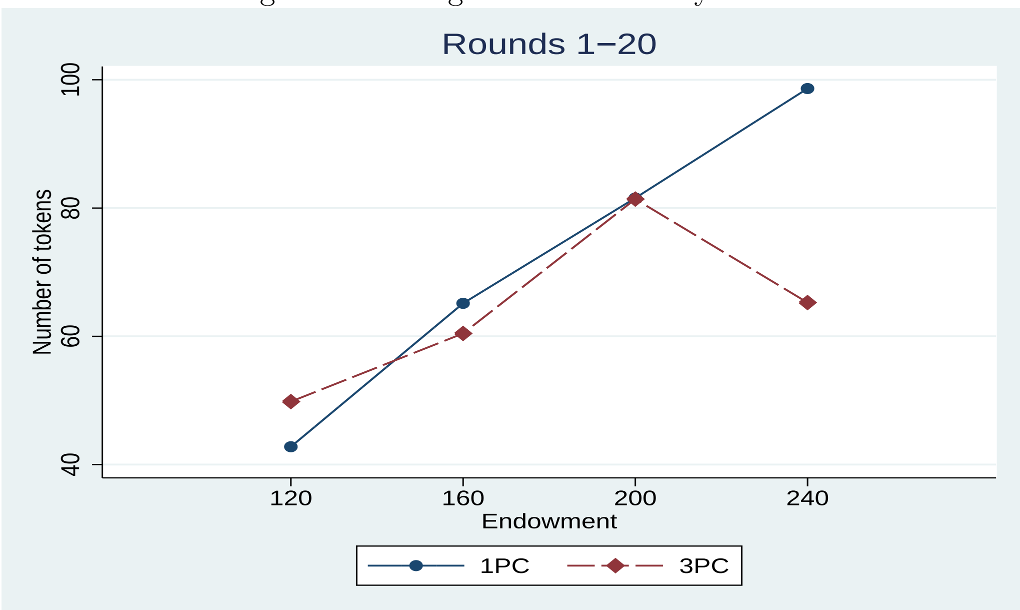

Figure 3: Average contributions, by endowment and rounds

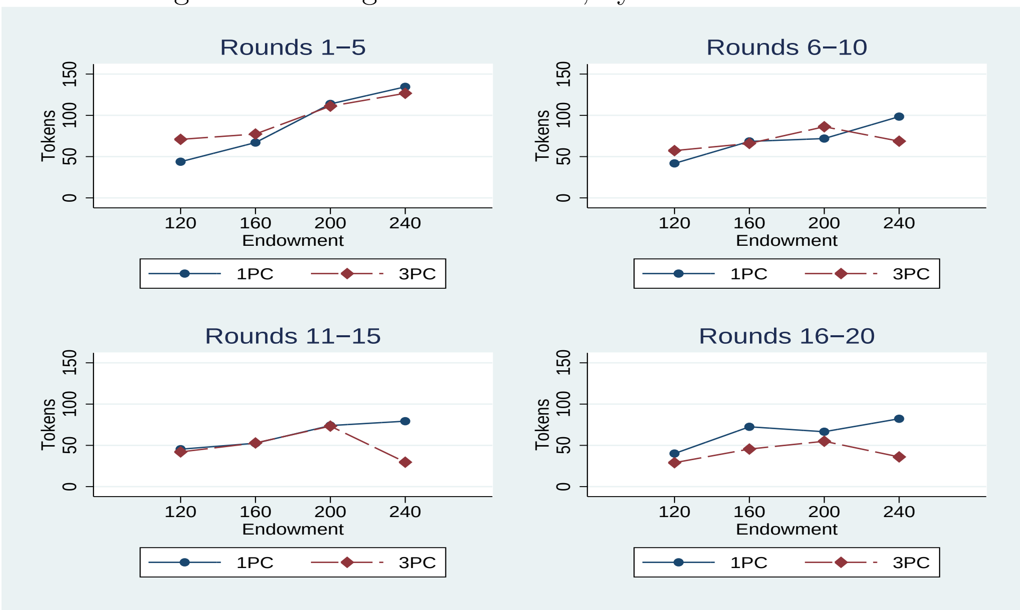

Figure 4: Empirical and predicted contributions, by endowment

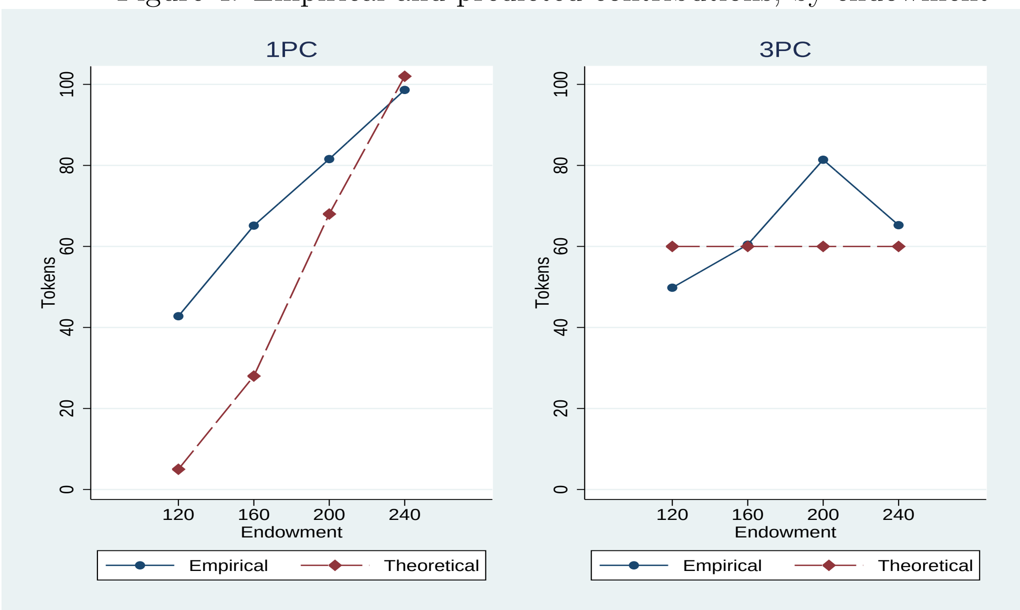

Figure 5: Empirical and predicted contributions over rounds: one prize

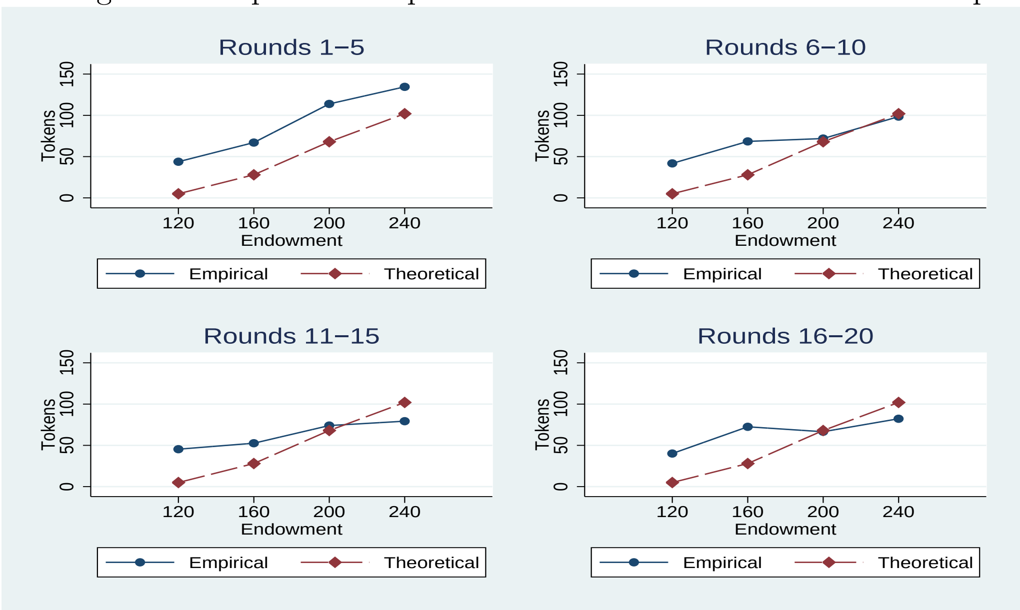

Figure 6: Empirical and predicted contributions over rounds: three prizes

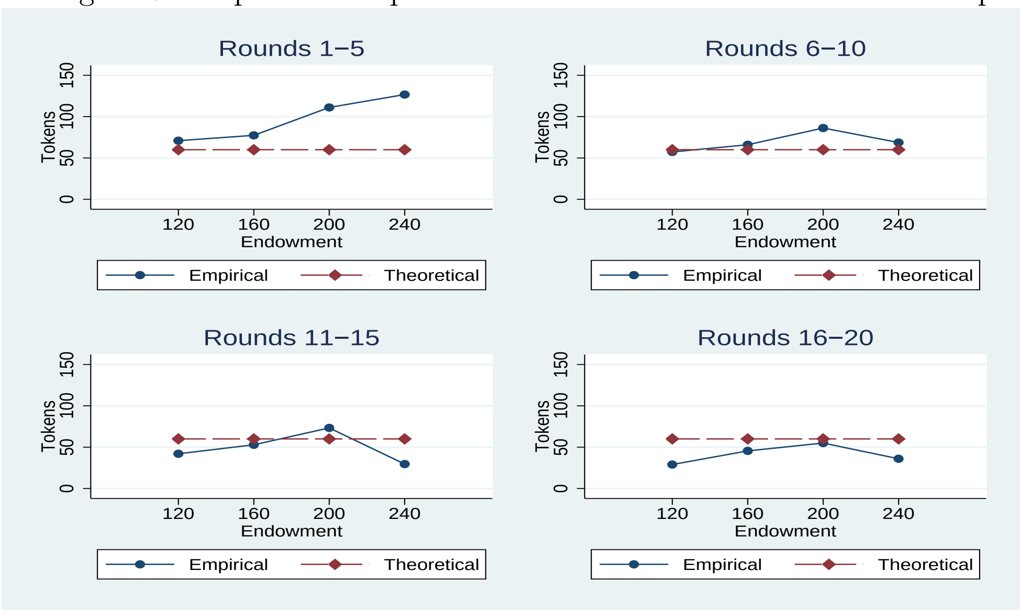

Figure 7: Distribution of contributions, by treatment

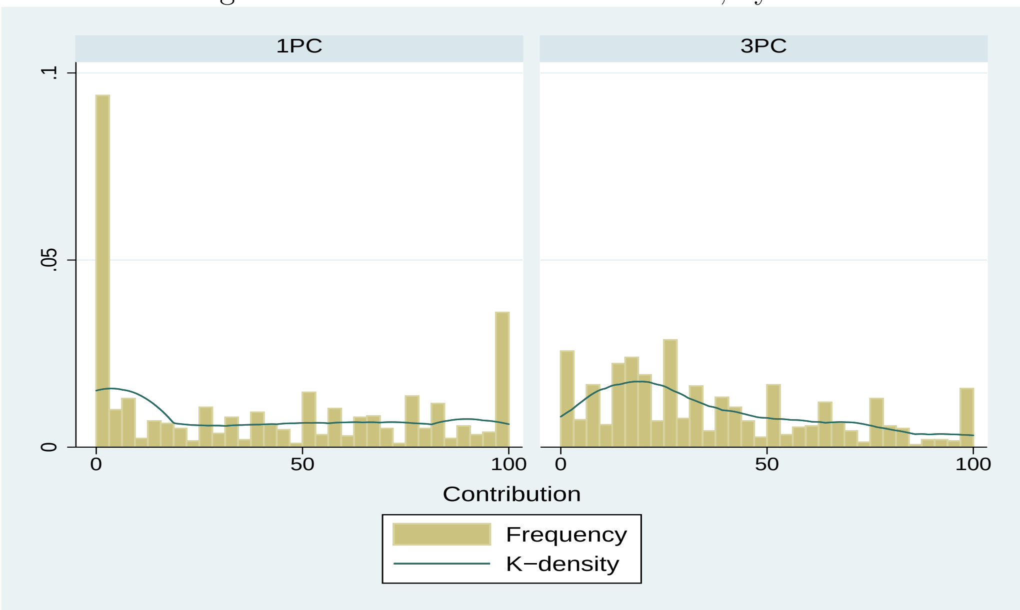

Figure 8: Cumulative distribution of contributions, by treatment

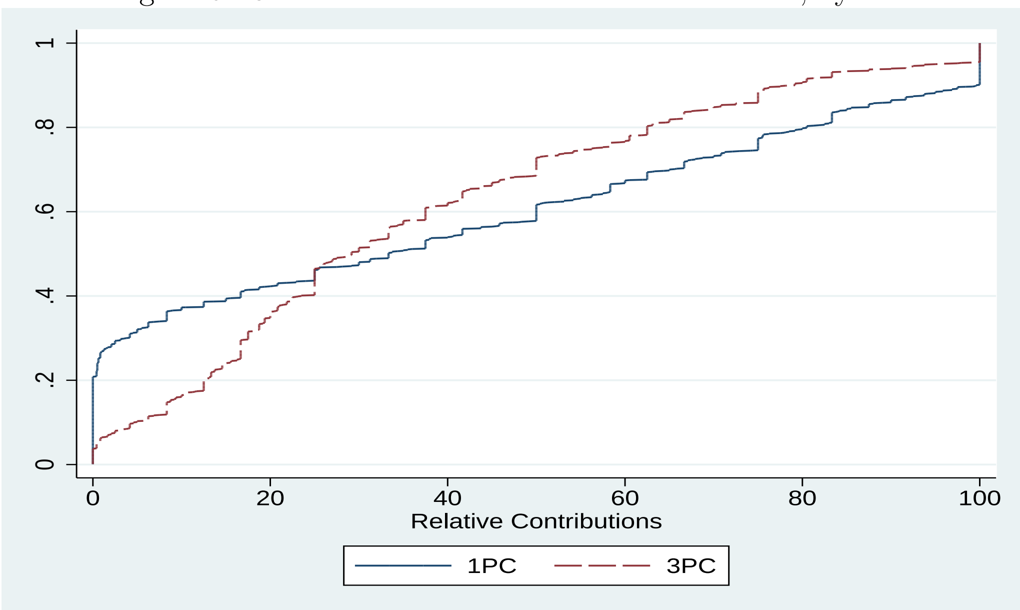

Figure 9: Distribution of contributions, by treatment and endowment

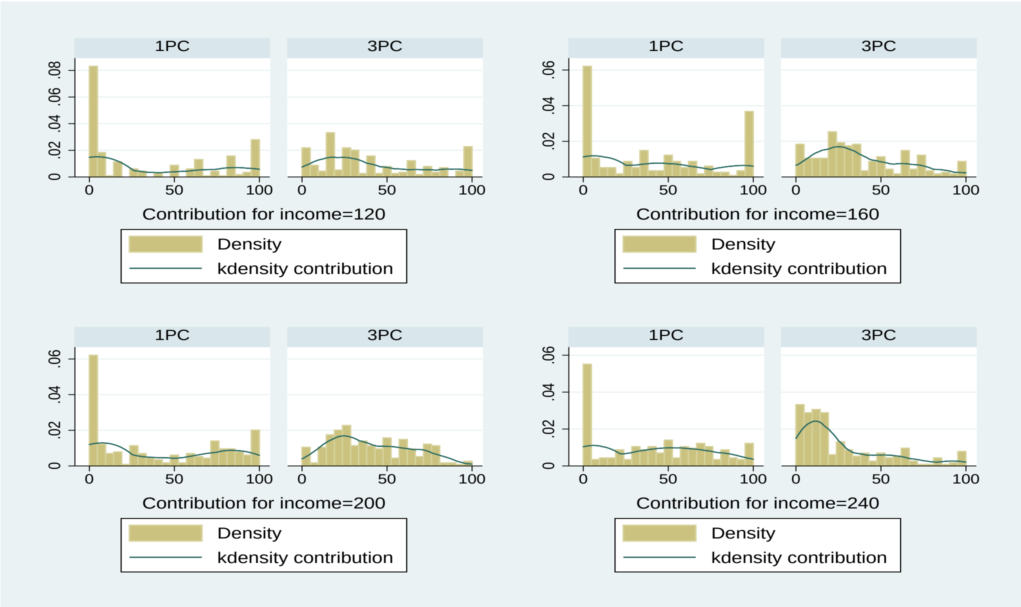

Figure 10: Cumulative distrib. of contributions, by treatment and endowment